Abstract

When expanding abroad, a multinational bank faces a trade-off between accessing a foreign country via cross border lending or a financial foreign direct investment, i.e. greenfield or acquisition entry. We analyze the entry mode choice of multinational banks and explicitly derive the entry mode pattern in the banking industry. Moreover, we show that in less developed banking markets, a trend towards cross border lending and acquisition entry exists. Greenfield entry prevails in more developed markets. Furthermore, we identify a tendency towards acquisition entry in small and towards greenfield entry in larger host countries.

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Proof of Lemma 1:

The marginal borrower between bank $A$ and $B_1$ and bank $A$ and $B_2$ is given by

$$x^CBL_{A,B_1} = \frac{1}{6} + \frac{r^CBL_A - r^CBL_{A,B_1}}{2t} \quad \text{and} \quad x^CBL_{A,B_2} = \frac{1}{6} + \frac{r^CBL_{A,B_2} - r^CBL_B}{2t}.$$  

It follows that the market share of banks can be expressed by $m\phi^CBL_A$, $m\phi^CBL_{B_1}$ and $m\phi^CBL_{B_2}$ with

$$\phi^CBL_A = \frac{1}{3} + \frac{r^CBL_A - 2r^CBL_{A,B_1}}{2t}, \quad \phi^CBL_{B_1} = \frac{1}{3} + \frac{r^CBL_{B_1} - 2r^CBL_{A,B_1}}{2t} \quad \text{and} \quad \phi^CBL_{B_2} = \frac{1}{3} + \frac{r^CBL_{B_2} - 2r^CBL_{A,B_2}}{2t}.$$  

Hence, banks’ profit functions are given by

$$\pi^CBL_A = \left[ \gamma \left( r^CBL_A - i_A \right) - (1 - \gamma) \left( 1 - \alpha \mu \delta_A \right) \left( 1 + i_A \right) \right] m\phi^CBL_A - FC^{CBL},$$

$$\pi^CBL_{B_1} = \left[ \gamma \left( r^CBL_{B_1} - i_B \right) - (1 - \gamma) \left( 1 - \delta_B \right) \left( 1 + i_B \right) \right] m\phi^CBL_{B_1},$$

$$\pi^CBL_{B_2} = \left[ \gamma \left( r^CBL_{B_2} - i_B \right) - (1 - \gamma) \left( 1 - \delta_B \right) \left( 1 + i_B \right) \right] m\phi^CBL_{B_2}.$$  

Proof of Proposition 1:

Note that $\frac{dr^CBL_A}{dr_A} = \frac{4m \alpha \mu (1 - \gamma) (1 + i_A) \phi^CBL_A}{5} > 0$ and $\frac{d^2\pi^CBL_A}{d\delta_A^2} = \frac{2m}{5\gamma} \left( \frac{3}{5} \alpha \mu (1 - \gamma) (1 + i_A) \right)^2 > 0$ so that $\pi^CBL$ is increasing and convex in $\delta_A$. Cross border lending is feasible for bank $A$ if $\pi^CBL_A > 0$.

Solving for bank $A$’s screening ability yields $\delta_A \geq \frac{5}{12} \sqrt{\frac{\gamma + \Delta}{m \alpha (1 - \gamma) (1 + i_A)}} \equiv \delta^CBL_A$.  

Note that since $\pi^CBL_A$ is increasing and convex in $\delta_A$ it follows that $\arg \min \{ \pi^CBL_A \} = -\frac{5}{6} \sqrt{\frac{\gamma + \Delta}{m \alpha (1 - \gamma) (1 + i_A)}} \leq 0$. Hence, $\frac{5}{6} \gamma + \Delta > 0$ must hold. We will refer to this condition as Condition (1): $\frac{5}{6} \gamma + \Delta > 0$.

Proof of Lemma 2:

Analogous to Proof of Lemma 1 we arrive at

$$r^CBL_A = \frac{1}{5\gamma} \left\{ \frac{5}{3} \gamma + 2i_B + 3i_A + (1 - \gamma) \left[ 5 - 2\delta_B (1 + i_B) - 3\mu \delta_A (1 + i_A) \right] \right\},$$

$$r^CBL_{B_1} = i^CBL_{B_2} = \frac{1}{5\gamma} \left\{ \frac{5}{3} \gamma + 4i_B + i_A + (1 - \gamma) \left[ 5 - 4\delta_B (1 + i_B) - \mu \delta_A (1 + i_A) \right] \right\} \equiv r^CBL_{B_2}.$$
\[ \pi^A_{GR} = mt\gamma \left( \phi^A_{GR} \right)^2 - F_{GR} \] and \[ \pi^B_{GR} = mt\gamma \left( \phi^B_{GR} \right)^2 = \pi^B_{GR}. \]

Proof of Proposition 2:

Note that \[ \frac{dx_{GR}^A}{dA} = \frac{4m\mu(1-\gamma)(1+i_A)\phi^A_{GR}}{5} > 0 \] and \[ \frac{d^2x_{GR}^A}{dA^2} = \frac{2m\gamma^2}{5}\mu(1-\gamma)(1+i_A)^2 > 0. \] Further, due to \( \phi_B > \phi_B \) and \( 0 < \alpha < 1 \) it holds that \[ \frac{dx_{GR}^A}{dA} > \frac{dx_{CBL}^A}{dA} \] and \[ \frac{d^2x_{GR}^A}{dA^2} > \frac{d^2x_{CBL}^A}{dA^2}. \] Since it also holds that \( \pi^C_{GR} \mid \delta_A = 0 = mt\gamma \left( \frac{2}{5}\mu(1+i_B) \right)^2 - F_{CBL} > \pi^A_{GR} \mid \delta_A = 0 = mt\gamma \left( \frac{2}{5\mu}(1+i_B) \right)^2 \), it follows that only one intersection between \( \pi^A_{GR} \) and \( \pi^C_{GR} \) is possible for \( \delta_A > 0 \). Greenfield entry is feasible for bank A in case of \( \pi^A_{GR} \geq \pi^C_{GR} \).

Solving for bank A’s screening ability yields \[ \delta_A \geq \frac{\sqrt{X_{GR} - \Delta - \frac{2}{5}t\gamma}}{\mu(1+\alpha)(1-\gamma)(1+i_A)} \equiv \delta^A_{GR} \] with \( X_{GR} \equiv (\Delta + \frac{5}{6}t\gamma)^2 + \frac{25\gamma(1+\alpha)(F_{GR} - F_{CBL})}{4m(1-\alpha)}. \)

Proof of Lemma 3:

Analogous to Proof of Lemma 1 we arrive at

\[ \tilde{\tau}^A_{AC} = \frac{1}{3\gamma^2} \left\{ \frac{3}{2} t\gamma + i_B + 2i_A + (1-\gamma) \left( 3 - \delta_B (1+i_B) - 2\delta_A (1+i_A) \right) \right\}, \]
\[ \tilde{\tau}^A_{BC} = \frac{1}{3\gamma^2} \left\{ \frac{3}{2} t\gamma + 2i_B + i_A + (1-\gamma) \left( 3 - 2\delta_B (1+i_B) - \delta_A (1+i_A) \right) \right\}, \]
\[ \pi^A_{AC} = mt\gamma \left( \tilde{\tau}^A_{AC} \right)^2 - P_{AC} - F_{AC} \] and \( \pi^A_{BC} = mt\gamma \left( \tilde{\tau}^A_{BC} \right)^2. \)

Proof of Proposition 3:

Derivation of Domestic Banks’ Profits with no Foreign Bank Entry

Analogous to Proof of Lemma 1 we arrive at

\[ \tilde{\tau}^B_{N1} = \frac{t\gamma + 2i_B + 2(1-\delta_B (1+i_B)) (1-\gamma)}{2t\gamma} \equiv \tilde{\tau}^B_{N1} \] and \( \pi^B_{N1} = \pi^B_{N2} = \frac{m\gamma}{4} \equiv \pi^B_{N2}. \)

Derivation of \( \delta^A_{AC} \)

Note, first, that it is useful to show that (1) \( \frac{dx_{AC}^A}{dA} > \frac{dx_{CBL}^A}{dA} \) in the range of \( \delta^C_{CBL} \leq \delta_A < \delta^C_{GR} \) and (2) \( \frac{dx_{AC}^A}{dA} > \frac{dx_{GR}^A}{dA} \) in the range of \( \delta_A \leq \delta_A < 1 \):

(1) proof of \( \frac{dx_{AC}^A}{dA} > \frac{dx_{CBL}^A}{dA} \) for \( \delta^C_{CBL} \leq \delta_A < \delta^C_{GR} \)

Note that \( \frac{dx_{AC}^A}{dA} = 2m(1-\gamma)(1+i_A) \left( \frac{1}{5} \phi_{AC} + \frac{1}{5} \alpha \mu \phi_B \right) > 0 \) and \( \frac{d^2x_{AC}^A}{dA} = 2m \left( \frac{1}{5} \gamma^2 (1-\gamma)^2 (1+i_A)^2 \right) > 0. \) Note also, beforehand, that by abstracting from exit of domestic banks, \( \tilde{\tau}_{AC}^A \leq \frac{2}{5} \) must hold due to the symmetric location of banks on the Salop circle. \( \tilde{\tau}_{AC}^A \leq \frac{2}{5} \) is equivalent to \( \frac{2}{6} t\gamma - \Delta - \mu (1-\gamma) (1+i_A) \delta_A \geq 0. \) We will use this condition further on and refer to it as Condition (1). \( \tilde{\tau}_{AC}^A \leq \frac{2}{5} \) is equivalent to \( \frac{2}{6} t\gamma - \Delta - \mu (1-\gamma) (1+i_A) \delta_A \geq 0. \) From Condition (2) follows a further useful condition which we will refer to as Condition (3): \( \frac{2}{6} t\gamma - \Delta > 0. \)
Note that \( \frac{\partial \pi^A_{\text{CBL}}}{\partial A} > \frac{\partial \pi^B_{\text{CBL}}}{\partial A} \) is equivalent to \( \frac{5}{6} t \gamma - \frac{5(9 \mu - 5)}{9(5-2\mu)} \Delta - \frac{5(9 \mu^2 \mu - 5)}{9(5-2\mu)} (1 - \gamma) (1 + i_A) \delta_A > 0 \) which is fulfilled due to Condition (2) since numerical simulations show that \(-1 < \frac{5(9 \mu - 5)}{9(5-2\mu)} < 1 \) and \( \frac{5(9 \mu^2 \mu - 5)}{9(5-2\mu)} < \mu \). Hence, \( \pi^A_{\text{CBL}} \) and \( \pi^B_{\text{CBL}} \) may intersect only once.

(2) proof of \( \frac{\partial \pi^A_{\text{CBL}}}{\partial A} > \frac{\partial \pi^B_{\text{CBL}}}{\partial A} \) for \( \delta^B_{\text{GR}} \leq \delta_A < 1 \)

Note that \( \frac{\partial \pi^A_{\text{CBL}}}{\partial A} = 2m (1 - \gamma) (1 + i_A) \left[ \frac{\gamma}{2} \delta_A^A + \frac{\gamma}{5} \delta_B^R \right] > 0 \) and \( \frac{\partial \pi^A_{\text{CBL}}}{\partial A} = \frac{2 m}{5} (1 - \gamma)^2 (1 + i_A)^2 \left( \frac{1}{6} - \frac{\mu^2}{25} \right) \)\( > 0 \). \( \frac{\partial \pi^A_{\text{CBL}}}{\partial A} > \frac{\partial \pi^B_{\text{GR}}}{\partial A} \) is equivalent to
\[
\frac{9}{5} t \gamma - \Delta - \frac{9 \mu - 5}{4} (1 - \gamma) (1 + i_A) \delta_A > 0 \]
which is fulfilled due to Condition (2) as \( \frac{9 \mu - 5}{4} < \mu \). Hence, \( \pi^A_{\text{CBL}} \) and \( \pi^B_{\text{GR}} \) may intersect only once.

As a consequence, \( \pi^A_{\text{CBL}} \) is increasing in \( \delta_A \) and jumps upwards twice due to the changing acquisition prices at \( \delta^B_{\text{CBL}} \) and \( \delta^B_{\text{GR}} \). Since, according to the above calculations, \( \pi^A_{\text{CBL}} \) is steeper than both \( \pi^B_{\text{CBL}} \) and \( \pi^B_{\text{GR}} \), in principle, four possible locations exist for \( \pi^A_{\text{CBL}} \). First, \( \pi^A_{\text{CBL}} \) could lie above \( \pi^B_{\text{CBL}} \) and \( \pi^B_{\text{GR}} \), thus eliminating cross border lending and greenfield entry from the entry mode pattern. Second, \( \pi^A_{\text{CBL}} \) could intersect with \( \pi^B_{\text{CBL}} \) which would exclude greenfield entry from the entry mode pattern. Third, and most interesting for us, \( \pi^A_{\text{CBL}} \) could intersect with \( \pi^B_{\text{GR}} \), allowing for the richest possible entry mode pattern. Fourth, \( \pi^A_{\text{CBL}} \) may be located below \( \pi^B_{\text{CBL}} \) and \( \pi^B_{\text{GR}} \), thus excluding acquisition entry from the entry mode pattern.

Since we concentrate throughout our analysis on the richest possible entry mode pattern, bank \( A \) chooses acquisition entry for \( \pi^A_{\text{CBL}} \geq \pi^B_{\text{GR}} \). Solving for bank \( A \)’s screening ability yields
\[
\delta_A \geq \frac{3 \gamma (5 - 2 \mu) - 2 \Delta (9 \mu - 5)}{2 \gamma (5 - 2 \mu) - 2 \Delta (9 \mu - 5)} (1 - \sqrt{X_{\text{AC}}} ) \equiv \delta^A_{\text{AC}} \text{ with } X_{\text{AC}} \equiv 1 + \frac{(9 \mu^2 - 5)[(5 t \gamma^2 + 36 t \gamma \Delta - 16 \Delta^2)]^{\frac{180 \mu}{5}}(F_{\text{GR}} - F_{\text{AC}})}{(3 \gamma (5 - 2 \mu) - 2 \Delta (9 \mu - 5))^2}.
\]

Proof of Proposition 5:

\[
\frac{d \delta^B_{\text{CBL}}}{d \mu} = -\frac{1}{\mu} \delta^B_{\text{CBL}} < 0 \text{ and } \frac{d \delta^B_{\text{GR}}}{d \mu} = -\frac{1}{\mu} \delta^B_{\text{GR}} < 0. \left| \frac{d \delta^B_{\text{CBL}}}{d \mu} \right| < \left| \frac{d \delta^B_{\text{GR}}}{d \mu} \right| \text{ holds as } \delta^B_{\text{CBL}} < \delta^B_{\text{GR}}.
\]

\[
\frac{d \delta^A_{\text{AC}}}{d \mu} = \frac{9}{4 \gamma} (5 - 2 \mu) - 2 \Delta (9 \mu - 5) [t \gamma + \Delta + 4 \mu \lambda (1 - \sqrt{X_{\text{AC}}}) - \frac{9}{4 \gamma} (5 - 2 \mu) - 2 \Delta (9 \mu - 5)]\left(1 - \sqrt{X_{\text{AC}}} \right) + \frac{\gamma}{(1 - \gamma) (1 + i_A) \sqrt{X_{\text{AC}}}} [5 t \gamma^2 + 36 t \gamma \Delta - 16 \Delta^2 + \frac{180 \mu}{5}] (F_{\text{GR}} - F_{\text{AC}})] \quad (3 \gamma (5 - 2 \mu) - 2 \Delta (9 \mu - 5))^2 \quad (3 \gamma (5 - 2 \mu) - 2 \Delta (9 \mu - 5))^2
\]

with \( \lambda \equiv \frac{1}{45} [3 t \gamma (5 - 2 \mu) - 2 \Delta (9 \mu - 5)] \).

Since \( \frac{(9 \mu^2 - 5)[(5 t \gamma^2 + 36 t \gamma \Delta - 16 \Delta^2)]^{\frac{180 \mu}{5}}(F_{\text{GR}} - F_{\text{AC}})}{(3 \gamma (5 - 2 \mu) - 2 \Delta (9 \mu - 5))^2} = X_{\text{AC}} - 1 = \sqrt{X_{\text{AC}}} - 1 \), \( \sqrt{X_{\text{AC}}} + 1 \), it follows

\[
\frac{d \delta^A_{\text{AC}}}{d \mu} = \frac{3 \gamma (5 - 2 \mu) - 2 \Delta (9 \mu - 5)}{2 \gamma (5 - 2 \mu) - 2 \Delta (9 \mu - 5)} \left[ \frac{1 - \sqrt{X_{\text{AC}}} \right] \left( \frac{2}{5 \lambda \sqrt{X_{\text{AC}}}} \right) \left[ \frac{3 \gamma (5 - 2 \mu) - 2 \Delta (9 \mu - 5)}{2 \gamma (5 - 2 \mu) - 2 \Delta (9 \mu - 5)} \left(1 - \sqrt{X_{\text{AC}}} \right) \mu (1 - \gamma) (1 + i_A) + \left( \frac{1}{45} t \gamma + \Delta \right) \right].
\]

Due to
\[
\frac{d \delta^A_{\text{AC}}}{d \mu} = \frac{3 \gamma (5 - 2 \mu) - 2 \Delta (9 \mu - 5)}{2 \gamma (5 - 2 \mu) - 2 \Delta (9 \mu - 5)} \left(1 - \sqrt{X_{\text{AC}}} \right) \equiv \delta^A_{\text{AC}}, \text{ this expression can be written as}
\]

\[
\frac{d \delta^A_{\text{AC}}}{d \mu} = \frac{4 \delta^A_{\text{AC}}}{25 \lambda \sqrt{X_{\text{AC}}}} \left[ \frac{5}{6} t \gamma + \Delta + \frac{3}{2} (\Delta + 5 \mu (1 - \gamma) (1 + i_A) \delta^A_{\text{AC}}) \right].
\]
Note, first, that $\frac{5}{6}t\gamma+\Delta>0$ due to Condition (1). Second, as we assume $\delta_A > \delta_B$ it must hold that $\phi_{AC} = \frac{1}{3}[\frac{5}{2}t\gamma - \Delta - (1 - \gamma)(1 + i_A)\delta_A] < \frac{1}{2}$. This is equivalent to $\Delta + (1 - \gamma)(1 + i_A)\delta_A > 0$. By assuming $\frac{5}{3}\mu \geq 1$, or, respectively, $\mu \geq 0.6$, we have that $\frac{3}{2}[\Delta + \frac{5}{3}\mu (1 - \gamma)(1 + i_A)\delta_A] \geq 0$. Third, $\lambda = \frac{1}{35}[3\gamma(5 - 2\mu) - 2\Delta(9\mu - 5)] > 0$ is equivalent to $\frac{5}{6}t\gamma - \frac{5(9\mu - 5)}{9(5 - 2\mu)}\Delta > 0$ which holds due to Condition (3) since $0 < \frac{5(9\mu - 5)}{9(5 - 2\mu)} < 1$. Hence, it holds that $\frac{d\delta_{AC}}{d\mu} > 0$.

Proof of Proposition 6:

$$\frac{d\delta_{AC}}{d\mu} = \frac{1 + i_B}{a(1 + i_A)} > 0 \quad \text{and} \quad \frac{d\delta_{GR}}{d\mu} = \frac{(1 - \gamma)(1 + i_B)\delta_{GR}}{X_{GR}} > 0$$

$$\frac{d\delta_{AC}}{d\mu} = \frac{(1 + i_B)(9\mu - 5)[1 - \sqrt{X_{AC}}]}{(1 + i_A)(9\mu - 5)\sqrt{X_{AC}}} + \frac{(1 + i_B)(9\mu - 5)[9\mu^2 - 5 + 180t\gamma(1 + i_A)\lambda\sqrt{X_{AC}}]}{(3\gamma(5 - 2\mu) - 2\Delta(9\mu - 5))^2}$$

With $\frac{9\mu^2 - 5}(5\gamma^2 + 36\gamma\Delta - 16\Delta^2) + \frac{180t\gamma(1 + i_A)\lambda\sqrt{X_{AC}}}{(3\gamma(5 - 2\mu) - 2\Delta(9\mu - 5))^2}$, we get

$$\frac{d\delta_{AC}}{d\mu} = \frac{2(1 + i_B)(1 - \gamma)(9\mu - 5)[9\mu^2 - 5 - 2\Delta(9\mu - 5)]}{45\lambda(1 + i_A)\lambda\sqrt{X_{AC}}} \left(1 - \sqrt{X_{AC}}\right)$$

Since $\frac{3\gamma(5 - 2\mu) - 2\Delta(9\mu - 5)}{2(9\mu^2 - 5)(1 - \gamma)(1 + i_A)} \left(1 - \sqrt{X_{AC}}\right) = \delta_{AC}$, it follows that

$$\frac{d\delta_{AC}}{d\mu} = \frac{2(1 + i_B)[4\left(\frac{5}{6}t\gamma - \Delta\right) - (1 - \gamma)(1 + i_A)(9\mu - 5)\delta_{AC}]}{45\lambda(1 + i_A)\lambda\sqrt{X_{AC}}}$$

Note that $4\left(\frac{5}{6}t\gamma - \Delta\right) - (1 - \gamma)(1 + i_A)(9\mu - 5)\delta_{AC} > 0$ is equivalent to $\frac{9}{8}t\gamma - \Delta - \frac{9\mu - 5}{4}(1 - \gamma)(1 + i_A)\delta_{AC} > 0$ which holds due to Condition (2) as $\frac{9\mu - 5}{4} < \mu$. Hence, $\frac{d\delta_{AC}}{d\mu} > 0$ holds.

Proof of Proposition 7:

$$\frac{d\delta_{BL}}{d\mu} = \frac{1 - (1 - \gamma)\delta_B}{a(1 - \gamma)(1 + i_A)} > 0 \quad \text{and} \quad \frac{d\delta_{GR}}{d\mu} = \frac{1 - (1 - \gamma)\delta_B\delta_{GR}}{X_{GR}} > 0.$$ Note that $\left|\frac{d\delta_{BL}}{d\mu}\right| > \left|\frac{d\delta_{GR}}{d\mu}\right|$ is equivalent to $\frac{1}{a} > -\frac{\frac{5}{6}t\gamma + \Delta}{\sqrt{X_{AC}}}$, which is fulfilled as $\frac{5}{6}t\gamma + \Delta > 0$ due to Condition (1).

$$\frac{d\delta_{AC}}{d\mu} = \frac{2\left(\frac{5}{6}t\gamma - \Delta\right)(1 - \gamma)(1 + i_B)}{a(1 - \gamma)(1 + i_A)} \left(\frac{3\gamma(5 - 2\mu) - 2\Delta(9\mu - 5)}{2(9\mu^2 - 5)(1 - \gamma)(1 + i_A)} \left(1 - \sqrt{X_{AC}}\right)\right)$$

With $\frac{9\mu^2 - 5}(5\gamma^2 + 36\gamma\Delta - 16\Delta^2) + \frac{180t\gamma(1 + i_A)\lambda\sqrt{X_{AC}}}{(3\gamma(5 - 2\mu) - 2\Delta(9\mu - 5))^2}$, we get

$$\frac{d\delta_{AC}}{d\mu} = \frac{2\left(\frac{5}{6}t\gamma - \Delta\right)(1 - \gamma)(1 + i_B)}{a(1 - \gamma)(1 + i_A)} \left(\frac{3\gamma(5 - 2\mu) - 2\Delta(9\mu - 5)}{2(9\mu^2 - 5)(1 - \gamma)(1 + i_A)} \left(1 - \sqrt{X_{AC}}\right)\right)$$

\footnote{Note that in order to keep our analysis tractable, we henceforth assume $\mu > 0.6$. We think this is justified since in that case, the foreign bank would not lose more than 40 percent of its screening efficiency due to soft information problems which seems reasonable.}
Since \(\frac{3\gamma(5-2\mu)-2\Delta(9\mu-5)}{2(9\mu^2-5)(1-\gamma)(1+i_A)} (1 - \sqrt{X_{AC}}) = \delta_A^{AC}\) it follows that
\[
\frac{d\delta_A^{AC}}{d\gamma} = \frac{-2(1-(1-\gamma)\delta_B)[4(\frac{2}{3}t\gamma - \Delta) - (1-\gamma)(1+i_A)(9\mu-5)\delta_A^{AC}]}{4\Delta\sqrt{X_{AC}}(1-\gamma)(1+i_A)}.
\]
Note that \(4(\frac{2}{3}t\gamma - \Delta) - (1-\gamma)(1+i_A)(9\mu-5)\delta_A^{AC} > 0\) is equivalent to \(\frac{9}{8}t\gamma - \Delta - \frac{9\mu-5}{4}(1-\gamma)(1+i_A)\delta_A^{AC} > 0\) which holds due to Condition (2) as \(\frac{9\mu-5}{4} < \mu\). Hence, \(\frac{d\delta_A^{AC}}{d\gamma} < 0\) holds.

**Proof of Proposition 8:**
\[
\frac{d\delta^{CBL}}{d\gamma} = \frac{5\gamma^2}{4\mu(1-\gamma)(1+i_A)} \left(\frac{2}{3} - \frac{\sqrt{F_{CBL}}}{m\gamma}\right)
\]
Note, first, that \(\delta_A^{CBL} < 1\) is equivalent to \(\sqrt{\frac{F_{CBL}}{m\gamma}} < \frac{1}{3}\). Second, as we abstract from exit of domestic banks it must hold that \(\tilde{\delta}^{CBL} \geq \frac{1}{3}\) which is equivalent to \(\frac{5}{6}t\gamma - \Delta - \alpha\mu (1-\gamma)(1+i_A)\delta_A \geq 0\). Accordingly, \(\frac{2\alpha\mu(1-\gamma)(1+i_A) + 2\Delta}{5\gamma} \leq \frac{1}{3}\) which is equivalent to \(\frac{5}{6}t\gamma - \Delta - \alpha\mu (1-\gamma)(1+i_A) \geq 0\) is fulfilled. Consequently, it must hold that \(\sqrt{\frac{F_{CBL}}{m\gamma}} < \frac{2}{3}\). Hence, it holds that \(\frac{d\delta^{CBL}}{d\gamma} > 0\).

\[
\frac{d\delta^{GR}}{d\gamma} = \frac{5\gamma^2}{6\sqrt{X_{GR}}} \left[\delta_A^{GR} - \frac{15(F_{GR}-F_{CBL})}{4\mu(1-\gamma)(1+i_A)}\right]
\]
Note that \(\frac{d\delta^{GR}}{d\gamma} > 0\) is equivalent to \(\delta_A^{GR} > \frac{15(F_{GR}-F_{CBL})}{4\mu(1-\gamma)(1+i_A)}\). Thus, it holds that \(\frac{d\delta^{GR}}{d\gamma} > 0\) for \(\delta_A^{GR} > \frac{15(F_{GR}-F_{CBL})}{4\mu(1-\gamma)(1+i_A)}\) and \(\frac{d\delta^{GR}}{d\gamma} < 0\) for \(\delta_A^{GR} < \frac{15(F_{GR}-F_{CBL})}{4\mu(1-\gamma)(1+i_A)}\).

\[
\frac{d\delta^{AC}}{d\gamma} = \frac{-\gamma^2(1-\frac{2}{3}\mu)[3\gamma(5-2\mu)-2\Delta(9\mu-5)]}{2(9\mu^2-5)(1-\gamma)(1+i_A)} (1 - \sqrt{X_{AC}}) = \frac{t^2\gamma(\frac{101}{360}t\gamma + \frac{1}{3}(\frac{2}{3}t\gamma - \Delta) - \frac{15(F_{GR}-F_{CBL})}{m\gamma})}{(1-\gamma)(1+i_A)\lambda\sqrt{X_{AC}}} - \frac{\gamma^2(1-\frac{2}{3}\mu)}{6(1-\gamma)(1+i_A)\sqrt{X_{AC}}}(\frac{m^2}{\gamma^2} - \frac{1}{3}) (\frac{5\gamma^2 + 36t\gamma - 16\Delta^2}{(3\gamma^2(5-2\mu)-2\Delta(9\mu-5)^2)})^2.
\]
With \(\frac{(9\mu^2-5)(5\gamma^2 + 36t\gamma - 16\Delta^2 + 180\mu)}{(3\gamma^2(5-2\mu)-2\Delta(9\mu-5)^2)} = X_{AC} - 1 = \sqrt{X_{AC} - 1} (\sqrt{X_{AC} + 1})\) we get
\[
\frac{d\delta^{AC}}{d\gamma} = \frac{-\gamma^2(1-\frac{2}{3}\mu)}{3\lambda} (3\gamma(5-2\mu)-2\Delta(9\mu-5))(1 - \sqrt{X_{AC}}) = \frac{t^2\gamma(\frac{101}{360}t\gamma + \frac{1}{3}(\frac{2}{3}t\gamma - \Delta) - \frac{15(F_{GR}-F_{CBL})}{m\gamma})}{(1-\gamma)(1+i_A)\lambda\sqrt{X_{AC}}} + \frac{\gamma^2(1-\frac{2}{3}\mu)}{3\lambda} (1+i_A)\sqrt{X_{AC}} \frac{3\gamma(5-2\mu)-2\Delta(9\mu-5)}{2(9\mu^2-5)(1-\gamma)(1+i_A)} (1 - \sqrt{X_{AC}}).
\]
Using \(3\gamma(5-2\mu)-2\Delta(9\mu-5)\) \(2(9\mu^2-5)(1-\gamma)(1+i_A)\) \(1 - \sqrt{X_{AC}}\) we arrive at
\[
\frac{d\delta^{AC}}{d\gamma} = \frac{t^2(1-\gamma)(1+i_A)(5-2\mu)\delta_A^{AC} + \frac{2}{3}t\gamma + 3\Delta + \frac{15}{16}(F_{GR}-F_{CBL})}{15(1-\gamma)(1+i_A)\lambda\sqrt{X_{AC}}}.
\]
We now prove that \(\frac{d\delta^{AC}}{d\gamma} > 0\). \(\frac{d\delta^{AC}}{d\gamma} > 0\) is equivalent to \(F_{AC} - F_{GR} < \frac{m}{15}t\) \(\Delta + 2\Delta + (5-2\mu)(1-\gamma)(1+i_A)\delta_A^{AC}\). Note, beforehand, that in case of acquisition entry \(\tilde{\delta}^{AC} \leq \frac{1}{2}\) must hold due to \(i_B - i_A > 0\), \(\delta_A > \delta_B\) and \(\mu = 0\). \(\tilde{\delta}^{AC} \leq \frac{1}{2}\) is equivalent to \(2\Delta + 2(1-\gamma)(1+i_A)\delta_A \geq 0\). As the minimum of \((5-2\mu)\) is 3, \(2\Delta + (5-2\mu)(1-\gamma)(1+i_A)\delta_A^{AC} > 0\) must then also hold.
As $\frac{5}{6} t\gamma + \Delta > 0$ due to Condition (3), it follows that the right hand side of the above inequality is positive. However, the left hand side of that expression is negative. As a consequence, the above inequality is fulfilled and it holds that $\frac{d\delta AC}{dt} > 0$.

**Proof of Proposition 9:**

\[
\frac{d\delta^{CBL}}{dm} = \frac{-5t\gamma}{4\alpha m^2 (1-\gamma)(1+i_A)} \frac{\sqrt{F_C^{BL}}}{m^2} < 0; \quad \frac{d\delta^{GR}}{dm} = \frac{-25t\gamma(F_{GR} - F_C^{BL})}{8\alpha m^2 (1-\gamma)(1+i_A)\sqrt{F_G^{GR}}} < 0; \quad \frac{d\delta^{AC}}{dm} = \frac{t\gamma(F_{GR} - F_AC)}{\lambda m^2 (1-\gamma)(1+i_A)\sqrt{F_A^{AC}}} > 0.
\]

**Proof of Lemma 4:**

(1) No Entry:

Consumer surplus is given by $vm\gamma + 0 \cdot m (1 - \gamma) - \tilde{r}_N^B m\gamma - 4m \frac{1}{M} \int_0 xtdx$. Producer surplus is given by $2\pi^N_B = mt\gamma \cdot \left(\frac{1}{2}\right)^2$. Rearranging yields $W_{NE} = m\gamma (v - i_B) - (1 - \gamma) (1 - \delta_B) (1 + i_B) - \left(\frac{1}{8}\right)^2$.

(2) Cross Border Lending:

Consumer surplus is given by $vm\gamma + 0 \cdot m (1 - \gamma) - (\tilde{r}_A^{CBL} - \phi_A^{CBL} + 2\tilde{r}_B^{CBL} - \phi_B^{CBL}) m\gamma - 2m \frac{1}{M} \int_0 xtdx + x_{A,B_1}^{CBL} \frac{1}{M} - x_{A,B_2}^{CBL} \frac{1}{M} \int_0 xtdx + \int_0 xtdx$. Producer surplus is given by $2\pi^{CBL}_B$. Rearranging yields

\[
W_{CBL} = m\left\{\gamma v - \frac{t}{12} - \frac{1}{3}\gamma (\tilde{r}_A^{CBL} + 2\tilde{r}_B^{CBL}) + \frac{1}{3} \gamma (\tilde{r}_A^{CBL} - \phi_A^{CBL})^2 + 2\left[\frac{1}{3} - \frac{1}{24} (\tilde{r}_A^{CBL} - \tilde{r}_B^{CBL})\right] \gamma (\tilde{r}_A^{CBL} - i_B) - (1 - \gamma) (1 - \delta_B) (1 + i_B)\right\}.
\]

(3) Greenfield Entry

Consumer surplus is given by $vm\gamma + 0 \cdot m (1 - \gamma) - (\tilde{r}_A^{GR} - \phi_A^{GR} + 2\tilde{r}_B^{GR} - \phi_B^{GR}) m\gamma - 2m \frac{1}{M} \int_0 xtdx + x_{A,B_1}^{GR} \frac{1}{M} - x_{A,B_2}^{GR} \frac{1}{M} \int_0 xtdx + \int_0 xtdx$. Producer surplus is given by $2\pi^{GR}_B$. Rearranging yields

\[
W_{GR} = m\left\{\gamma v - \frac{t}{12} - \frac{1}{3}\gamma (\tilde{r}_A^{GR} + 2\tilde{r}_B^{GR}) + \frac{1}{3} \gamma (\tilde{r}_A^{GR} - \phi_A^{GR})^2 + 2\left[\frac{1}{3} - \frac{1}{24} (\tilde{r}_B^{GR} - \tilde{r}_A^{GR})\right] \gamma (\tilde{r}_B^{GR} - i_B) - (1 - \gamma) (1 - \delta_B) (1 + i_B)\right\}.
\]

\[\text{For our analysis to be interesting, we assume } W_{NE} > 0.\]
(4) Entry via Acquisition

Consumer surplus is given by \( v m \gamma + 0 \cdot m (1 - \gamma) - (\tilde{r}^A \tilde{\phi}_{A} + \tilde{r}^B \tilde{\phi}_{B}) m \gamma - 2 m \left( \int_{0}^{x_{A,B}^{AC}} \frac{1}{2} x_{A,B}^{AC} \right) \int_{0}^{x_{A,B}^{AC}} xdx \). Producer surplus is given by \( \pi^{AC} \) and the acquisition price amounts to \( \pi^{GR} \).

Rearranging yields

\[
W_{AC} = m \{ v - \tilde{r}^A (\frac{1}{2} + \frac{1}{t} (\tilde{r}^B - \tilde{r}^A)) - (\frac{1}{2} - \frac{1}{t} (\tilde{r}^B - \tilde{r}^A)) i_B \} - (\frac{1}{2} - \frac{1}{t} (\tilde{r}^B - \tilde{r}^A)) (1 - \gamma) (1 - \delta_B) (1 + i_B) - \frac{1}{2} \left[ \frac{1}{t} + \frac{1}{7} (\tilde{r}^B - \tilde{r}^A)^2 \right] + \frac{1}{25t} \left[ \frac{5}{3} t \gamma - \Delta - \mu (1 - \gamma) (1 + i_A) \delta_A \right]^2 \}.
\]

Proof of Proposition 10:

Shape of Welfare Functions:

\[
\frac{dW_{NE}}{d\delta_A} = 0
\]

\[
\frac{dW_{CBL}}{d\delta_A} \mid_{\delta_A = 0} = \frac{m \mu (1 - \gamma) (1 + i_A)}{75t \gamma^2} \left[ 5 t \gamma^2 + 12 (3\gamma - 1) (i_B - i_A) \right] > 0
\]

\[
\frac{d^2W_{CBL}}{d\delta_A^2} = \frac{m (1 - \gamma) (1 - 3\gamma^2)}{t} (2 \mu (1 + i_A)) ^2 > 0
\]

\[
\frac{dW_{GR}}{d\delta_A} \mid_{\delta_A = 0} = \frac{m \mu (1 - \gamma) (1 + i_A)}{75t \gamma^2} \left[ 5 t \gamma^2 + 12 (3\gamma - 1) (i_B - i_A) \right] > 0
\]

\[
\frac{d^2W_{GR}}{d\delta_A^2} = \frac{m (1 - \gamma) (1 - 3\gamma^2)}{t} (2 \mu (1 + i_A)) ^2 > 0
\]

\[
\frac{dW_{AC}}{d\delta_A} \mid_{\delta_A = 0} = \frac{m (1 + i_A) (1 - \gamma)}{450t \gamma^2} \left[ 15 t \gamma^2 (5 - 4\mu) + 2 (100\gamma + 18\mu \gamma - 25) (i_B - i_A) \right] > 0
\]

\[
\frac{d^2W_{AC}}{d\delta_A^2} = \frac{m (1 + i_A) (1 - \gamma)}{15 \gamma^2} \left( \frac{(1 - \gamma) (1 + i_A)}{15\gamma} \right)^2 > 0
\]

Note that \( 4\gamma - 1 - 3\gamma^2 > 0 \) as well as \( 18\mu ^2 + 100\gamma - 25 > 0 \) hold for \( \gamma > 0.5 \) which we will assume henceforth. We find, first, that \( W_{NE} \) is independent of \( \delta_A \). Second, \( W_{CBL}, W_{GR}, \) and \( W_{AC} \) are quadratic, increasing, and convex functions in \( \delta_A \) with arg min \( (W_{CBL}) < 0, \) arg min \( (W_{GR}) < 0, \) and arg min \( (W_{AC}) < 0 \) since the second order conditions with respect to \( \delta_A \) are positive and since the first order conditions with respect to \( \delta_A \) at \( \delta_A = 0 \) are positive as well. Next, we show that \( W_{GR} > W_{CBL} \) always holds. \( W_{GR} - W_{CBL} > 0 \) is equivalent to

\[
\frac{2}{25t} m \mu \delta_A (1 - \gamma) (1 + i_A) (1 - \alpha) + \frac{3}{5} t \gamma + 2 \left( \frac{3\gamma - 1}{2} \right) \left[ \Delta + \frac{1}{2} \mu \delta_A (1 - \gamma) (1 + i_A) (1 + \alpha) \right] > 0
\]

which is fulfilled if we assume \( \Delta > 0.3 \). Hence, we only need to calculate the intersection points of \( W_{NE}, W_{GR}, \) and \( W_{AC} \).

\[3\text{Regarding the welfare analysis, we only look at host banking markets on a rather low financial development stage and assume henceforth } \Delta = i_B - i_A - (1 - \gamma) (1 + i_B) \delta_B > 0. \text{ This expression is the more likely fulfilled the larger is the difference in refinancing conditions of foreign and host country banks and the lower is the screening ability of host country banks. This limitation seems justified as the entry of foreign banks into financially very well developed countries is, in general, not very much regulated.} \]
(a) Intersection between $W_{NE}$ and $W_{GR}$

Note that $W_{GR} - W_{NE} > 0$ is equivalent to

$$\frac{m}{1800 \mu^2} \{144 \mu^2 (3 \gamma - 1) (1 - \gamma)^2 (1 + i_A)^2 (\delta_A)^2 + 24 \mu (1 - \gamma) (1 + i_A) [5t \gamma^2 + 12(3 \gamma - 1)\Delta] \delta_A - 5t \gamma^2 [5t (8 \gamma - 3) - 24 \Delta] + 144(3 \gamma - 1)\Delta^2\} > 0.$$  

Solving for $\delta_A$ yields $\delta_A < \frac{\frac{5}{6}t \gamma (\sqrt{25 \gamma^2 - 17 \gamma + 3 - \gamma}) - (3 \gamma - 1)\Delta}{\mu(1 - \gamma)(3 \gamma - 1)(1 + i_A)}$ and $\delta_A > \frac{\frac{5}{6}t \gamma (\sqrt{25 \gamma^2 - 17 \gamma + 3 - \gamma}) - (3 \gamma - 1)\Delta}{\mu(1 - \gamma)(3 \gamma - 1)(1 + i_A)}$.

Since, as derived above, $\arg \min_{\delta_A \in \mathbb{R}} (W_{GR}) < 0$, $\frac{dW_{NE}}{d\delta_A} = 0$ and $\frac{dW_{GR}}{d\delta_A} |_{\delta_A = 0} > 0$, only one intersection between $W_{NE}$ and $W_{GR}$ for $0 < \delta_A < 1$ is possible so that we only need to consider the intersection point $\delta_A = \frac{\frac{5}{6}t \gamma (\sqrt{25 \gamma^2 - 17 \gamma + 3 - \gamma}) - (3 \gamma - 1)\Delta}{\mu(1 - \gamma)(3 \gamma - 1)(1 + i_A)}$.

Hence, $W_{GR} > W_{NE}$ is equivalent to $\delta_A > \frac{\frac{5}{6}t \gamma (\sqrt{25 \gamma^2 - 17 \gamma + 3 - \gamma}) - (3 \gamma - 1)\Delta}{\mu(1 - \gamma)(3 \gamma - 1)(1 + i_A)} \equiv \delta_{GR}$.

(b) Intersection between $W_{GR}$ and $W_{AC}$

Note that $W_{AC} - W_{GR} > 0$ is equivalent to

$$\frac{m}{1800 \mu^2} \{4 (1 - \gamma)^2 (1 + i_A)^2 (100 \gamma - 25 + 36 \mu^2 - 90 \mu^2 \gamma) (\delta_A)^2 - 4 (1 - \gamma) (1 + i_A) [(15 t \gamma^2 (6 \mu - 5) - 2 (100 \gamma - 25 + 36 \mu - 90 \gamma \mu) \Delta)] \delta_A + 4 (10 \gamma + 11) \Delta^2 - 25 t \gamma^2 (2 \gamma + 3) - 60 t \gamma^2 \Delta\} > 0.$$  

Solving for $\delta_A$ yields $\delta_A < x_{AC}^W - \frac{5 \sqrt{X_{AC}^W}}{2 (1 - \gamma)(1 + i_A)(25 (4 \gamma - 1) - 18 \mu^2 (5 \gamma - 2))}$ and $\delta_A > x_{AC}^W + \frac{5 \sqrt{X_{AC}^W}}{2 (1 - \gamma)(1 + i_A)(25 (4 \gamma - 1) - 18 \mu^2 (5 \gamma - 2))}$

with $x_{AC}^W = \frac{\frac{5}{6}t \gamma (6 \mu - 5) \gamma - 25 (4 \gamma - 1) - 18 \mu (5 \gamma - 2) \Delta}{(1 - \gamma)(1 + i_A)(25 (4 \gamma - 1) - 18 \mu^2 (5 \gamma - 2))}$ and $X_{AC}^W = t^2 \{t [25 (10 \gamma + 17 \gamma^2 - 3) + 18 \mu (6 \mu - 30 \gamma^2 - 11 \gamma \mu + 8 \gamma^2 \mu)] + 72 (1 - \mu) (5 \gamma (4 - 3 \mu) + 6 \mu - 5) \Delta\} + 72 (1 - \mu) (4 \gamma - 1) (5 \gamma - 2) \Delta^2$.

In principle, two intersections of $W_{AC}$ and $W_{GR}$ for $\delta_A > 0$ are possible. However, remember that $\frac{dW_{AC}}{d\delta_A} |_{\delta_A = 0} > W_{AC} |_{\delta_A = 0}$ we show that we only need to consider the upper intersection point $x_{AC}^W + \frac{5 \sqrt{X_{AC}^W}}{2 (1 - \gamma)(1 + i_A)(25 (4 \gamma - 1) - 18 \mu^2 (5 \gamma - 2))}$. Note that $W_{GR} |_{\delta_A = 0} - W_{AC} |_{\delta_A = 0} = m \frac{25 t^2 \gamma^2 (2 \gamma + 3) - 4 \Delta^2 (10 \gamma + 11) + 60 t^2 \gamma^2 \Delta}{1800 \mu^2}$. Further, $25 t^2 \gamma^2 (2 \gamma + 3) - 4 \Delta^2 (10 \gamma + 11) > 0$ is equivalent to $\frac{5}{6}t \gamma - \frac{1}{3} \sqrt{\frac{(10 \gamma + 11)}{(2 \gamma + 3)}} \Delta > 0$. Numerical solutions show that $0 < \frac{1}{3} \sqrt{\frac{(10 \gamma + 11)}{(2 \gamma + 3)}} < 1$ so that Condition (2) is fulfilled and, hence, it must hold that $W_{GR} |_{\delta_A = 0} > W_{AC} |_{\delta_A = 0}$. It follows that $W_{AC} > W_{GR}$ holds for $\delta_A > x_{AC}^W + \frac{5 \sqrt{X_{AC}^W}}{2 (1 - \gamma)(1 + i_A)(25 (4 \gamma - 1) - 18 \mu^2 (5 \gamma - 2))} \equiv \delta_{AC}$.

As we can abstract from the lower threshold, it must further hold that $\frac{dW_{AC}}{d\delta_A} |_{\delta_A = \delta_{AC}} > \frac{dW_{GR}}{d\delta_A} |_{\delta_A = \delta_{AC}}$.  

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(c) Intersection of $W_{NE}$ and $W_{AC}$ and proof of $\delta^{GR} < \delta^{AC}$

Note, first, that it is useful to show that $\delta^{GR} < \delta^{AC}$. $\delta^{GR} < \delta^{AC}$ is equivalent to

$$\frac{5}{25(4\gamma - 1)^{3} - 18\mu^{2}(5\gamma - 2)^{2}} \left( t^{2} \left\{ 25(10\gamma + 17\gamma^{2} - 3) + 18\mu (6\mu - 30\mu^{2} - 11\gamma \mu + 8\gamma^{2} \mu) \right\} + 72(1 - \mu) \left[ 5\gamma (4 - 3\mu) + 6\mu - 5 \right] \Delta \right) + 72(1 - \mu)^{2} (4\gamma - 1) (5\gamma - 2) \Delta^{2} \right]^{\frac{1}{2}} > \frac{2\mu(3\gamma - 1)25(4\gamma - 1) - 18\mu^{2}(5\gamma - 2)^{2}}{\left\{ -\gamma (90\mu + 100\gamma + 234\mu^{2} \gamma - 270\mu \gamma - 72\mu^{2} - 25) + \text{[25 (4\gamma - 1) - 18\mu}^{2} (5\gamma - 2) \right] \sqrt{-17\gamma + 25\gamma^{2} + 3} \}$$

Numerical simulations show that $25 (4\gamma - 1) - 18\mu^{2} (5\gamma - 2) > 0$ so that the left hand side of this expression is clearly positive. The right hand side is also positive as numerical simulations show that $\left\{ -\gamma (90\mu + 100\gamma + 234\mu^{2} \gamma - 270\mu \gamma - 72\mu^{2} - 25) + \text{[25 (4\gamma - 1) - 18\mu}^{2} (5\gamma - 2) \right] \sqrt{-17\gamma + 25\gamma^{2} + 3} \}^{2} > \frac{\text{[25 (4\gamma - 1) - 18\mu}^{2} (5\gamma - 2) \right] \sqrt{-17\gamma + 25\gamma^{2} + 3} \} + \frac{t^{2} \gamma^{2}}{\gamma [18\gamma \mu (-10\gamma + 17\mu + 30\gamma^{2} - 95\gamma \mu + 130\gamma^{2} \mu) - 25 (4\gamma - 1) (-17\gamma + 26\gamma^{2} + 3) + 2\gamma [25 (4\gamma - 1) + 18\mu (13\gamma \mu + 5 - 15\gamma - 4\mu)] \sqrt{-17\gamma + 25\gamma^{2} + 3}]} > 0$

Numerical simulations show that $[18\mu (3\gamma - 1) - 5 (4\gamma - 1)] > 0$, $-1 < \frac{5(4\gamma - 1)(3\gamma - 1)(1 - \mu)}{\gamma [18\mu (3\gamma - 1) - 5 (4\gamma - 1)] + 5(4\gamma - 1) \sqrt{-17\gamma + 25\gamma^{2} + 3}} < 1$ and

$$\left[ 18\gamma \mu (-10\gamma + 17\mu + 30\gamma^{2} - 95\gamma \mu + 130\gamma^{2} \mu) - 25 (4\gamma - 1) (-17\gamma + 26\gamma^{2} + 3) + 2\gamma [25 (4\gamma - 1) + 18\mu (13\gamma \mu + 5 - 15\gamma - 4\mu)] \sqrt{-17\gamma + 25\gamma^{2} + 3} \right] > 0 \text{ for } \mu > 0.75.\text{\footnote{In order to keep our analysis tractable, we henceforth assume } \mu > 0.75.}$$

Hence, the whole expression is positive since due to Condition (3)

$$\left[ \frac{5}{6} t^{2} \gamma - \frac{5(4\gamma - 1)(3\gamma - 1)(1 - \mu)}{\gamma [18\mu (3\gamma - 1) - 5 (4\gamma - 1)] + 5(4\gamma - 1) \sqrt{-17\gamma + 25\gamma^{2} + 3}} \Delta \right] > 0.$$  

The above calculations show that $\delta^{GR} < \delta^{AC}$. Remember that we also found that $\text{arg min}_{W_{GR}} < 0$, $\frac{dW_{GR}}{d\delta_{A}} |_{\delta_{A} = 0} > 0$, $\text{arg min}_{W_{AC}} < 0$, $\frac{dW_{AC}}{d\delta_{A}} |_{\delta_{A} = 0} > 0$, and $\frac{dW_{AC}}{d\delta_{A}} |_{\delta_{A} = \delta_{A}^{TP}} > \frac{dW_{GR}}{d\delta_{A}} |_{\delta_{A} = \delta_{A}^{TP}}$. Hence, we can neglect the intersection point between $W_{NE}$ and $W_{AC}$, since the policy maker would always prefer greenfield or acquisition entry to the right hand side of this point.

(d) Entry Mode Pattern Preferred by the Social Planner

It follows from the analysis above that the entry mode pattern the social planner prefers is increasing in the screening ability of the foreign bank: no entry - greenfield entry - acquisition entry. Again, one or both entry modes could drop out of the pattern depending on the parameter constellations, but the sequence of the pattern can never be different.
Proof of Proposition 11:

(1) Proof of $\delta_A^{CBL} < \delta_A^{GR} < \delta_W^{GR}$

Note that the policy maker cannot require that the foreign bank enters via a de novo investment if the foreign bank makes losses in case of greenfield entry. Hence, greenfield entry is only possible for $\pi_A^{GR} \geq 0$ which is equivalent to $\delta_A \geq \frac{1}{\mu (1-\gamma)(1+i_A)} \left( \frac{5}{8} \sqrt{\frac{\mu F_{GR}}{m}} - \frac{5}{8} t \gamma - \Delta \right) \equiv \delta_A^{GR}$. Note that $\delta_A^{GR} > \delta_A^{CBL}$ holds.

(2) Proof of $\delta_W^{AC} > \delta_A^{AC}$

Note that $\delta_W^{AC} > \delta_A^{AC}$ is equivalent to

$$\frac{6}{5} [25 (15 \gamma + 2 \mu - 2 \gamma \mu - 5) + 9 \mu^2 (-25 \gamma - 8 \mu - 10 \gamma \mu + 20)]$$

$$\left[ \frac{5}{6} t \gamma - \frac{25 \mu (1-\mu)(10 \gamma -1) \Delta}{25 \mu (1-\mu)(10 \gamma -1) \Delta + 9 \mu^2 (20 - 25 \gamma - 8 \mu - 10 \gamma \mu)} \right] < 15 \lambda [25 (4 \gamma - 1) - 18 \mu^2 (5 \gamma - 2)] \sqrt{X_{AC}} + \frac{5}{9} (9 \mu^2 - 5) \sqrt{X_{AC}}.$$

The right hand side of this expression is positive since $25 (4 \gamma - 1) - 18 \mu^2 (5 \gamma - 2) > 0$ as shown before. The left hand side is positive if $25 (15 \gamma + 2 \mu - 2 \gamma \mu - 5) + 9 \mu^2 (20 - 25 \gamma - 8 \mu - 10 \gamma \mu) > 0$ and $\frac{5}{6} t \gamma - \frac{25 \mu (1-\mu)(10 \gamma -1) \Delta}{25 \mu (1-\mu)(10 \gamma -1) \Delta + 9 \mu^2 (20 - 25 \gamma - 8 \mu - 10 \gamma \mu)} > 0$. Numerical simulations show that the first expression is fulfilled and that $-1 < \frac{30 \mu (1-\mu)(10 \gamma -1)}{25 \mu (1-\mu)(10 \gamma -1) \Delta + 9 \mu^2 (20 - 25 \gamma - 8 \mu - 10 \gamma \mu)} < 1$ holds, which, in turn, guarantees that Condition (3) is fulfilled and, accordingly, $\frac{5}{6} t \gamma - \frac{15 (10 \gamma -3)(1-\mu)}{2 (13 \mu^2 + 10 - 36 \mu - 70 \gamma \Delta)} > 0$ holds. Squaring both sides and rearranging yields

$$\frac{4 \gamma^2 (400 + 216 \mu^2 \gamma - 720 \mu \gamma - 171 \mu^2 + 100)}{25 (4 \gamma - 1) - 18 \mu^2 (5 \gamma - 2)} - 12 \Delta (1-\mu) \left[ 27 \Delta - 27 \mu \Delta - 90 \gamma \Delta - 70 \gamma^2 + 10 \gamma t + 90 \mu \gamma \Delta + 135 \mu t \gamma^2 - 36 \mu \gamma t \right]$$

$$\frac{36 \gamma^2}{m} (F_{GR} - F_{AC}) + \frac{90 \lambda \sqrt{X_{AC}}}{25 (4 \gamma - 1) - 18 \mu^2 (5 \gamma - 2)},$$

The right hand side of this inequality is positive. The left hand side, however, is negative. To see this, note, first, that $(400 + 216 \mu^2 \gamma - 720 \mu \gamma - 171 \mu^2 + 100) < 0$. Second, $27 \Delta - 27 \mu \Delta - 90 \gamma \Delta - 70 \gamma^2 + 10 \gamma t + 90 \mu \gamma \Delta + 135 \mu t \gamma^2 - 36 \mu \gamma t > 0$ is equivalent to $\frac{5}{6} t \gamma - \frac{15 (10 \gamma -3)(1-\mu)}{2 (13 \mu^2 + 10 - 36 \mu - 70 \gamma \Delta)} > 0$. Note that $-1 < \frac{15 (10 \gamma -3)(1-\mu)}{2 (13 \mu^2 + 10 - 36 \mu - 70 \gamma \Delta)} < 1$ holds for not too small $(\gamma, \mu)$ combinations; then, $135 \mu^2 + 10 - 36 \mu - 70 \gamma > 0$ also holds. Accordingly, Condition (3) is fulfilled and it follows that $\frac{6}{5} (135 \mu^2 + 10 - 36 \mu - 70 \gamma) \left( \frac{5}{6} t \gamma - \frac{15 (10 \gamma -3)(1-\mu)}{2 (13 \mu^2 + 10 - 36 \mu - 70 \gamma \Delta)} > 0.$

As a consequence, the left hand side of the above expression is negative. Hence, the above expression is true and, consequently, it holds that $\delta_W^{AC} > \delta_A^{AC}$.

(3) Proof of $\frac{d \delta_W^{GR}}{d t} < 0$

$$\frac{d \delta_W^{GR}}{d t} = -\frac{5 \gamma^2}{12 \mu (1-\gamma)(3 \gamma -1)(1+i_A)} \left( \sqrt{25 \gamma^2 - 17 \gamma + 3} - \gamma \right) < 0.$$

\[5\text{To keep our analysis tractable, we assume } \mu > \frac{5}{3} \frac{5 \gamma - 13}{13 \gamma - 35}, \text{ which is slightly larger than 0.75.}\]