Appendix to "Technology Transfer and Spillovers in International Joint Ventures"

Thomas Müller* Monika Schnitzer†

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*Bundestanstalt für Finanzdienstleistungsaufsicht (BaFin), International/Financial Markets Department, Georg-von-Boeselager-Str. 25, 53117 Bonn, Germany, Tel.: +49 228 4108 3732, Fax.: +49 228 4108 63732, e-mail: Thomas.Mueller3@bafin.de

†Department of Economics,University of Munich, Akademiestr. 1/III, 80799 Munich, Germany, Tel.: +49-89-2180 2217, Fax.: +49-89-2180 2767, e-mail: schnitzer@lrz.uni-muenchen.de

Proof of Result 1

MNE maximizes

$$U_{MNE} = q\alpha R - q(1-\alpha)\beta S - \alpha K(q).$$

Given the assumptions on $K(q)$ the optimal level of investment q is uniquely characterized by the following first order condition:

$$K'(q) = R - \frac{1-\alpha}{\alpha}\beta S.$$

Using the implicit function theorem we can show that

$$\frac{dq}{dS} = -\frac{-\frac{1-\alpha}{\alpha}\beta}{-K''(q)} = -\frac{(1-\alpha)\beta}{\alpha K''(q)} < 0$$

$$\frac{dq}{d\alpha} = -\frac{-\frac{(-\alpha-(1-\alpha))}{\alpha^2}\beta S}{-K''(q)} = -\frac{\beta S}{\alpha^2 K''(q)} > 0$$

Using these results we can show that

$$\begin{aligned}
\frac{dU_{MNE}}{dS} &= \frac{dq}{dS}\alpha R - \frac{dq}{dS}(1-\alpha)\beta S - q(1-\alpha)\beta - \alpha\frac{dq}{dS}K'(q) \\
&= \frac{dq}{dS}\underbrace{[\alpha R - (1-\alpha)\beta S - \alpha K'(q)]}_{=0} - q(1-\alpha)\beta \\
&= -q(1-\alpha)\beta < 0
\end{aligned}$$

Furthermore

$$\begin{aligned}
\frac{dU_{HC}}{dS} &= \frac{dq}{dS}(1-\alpha)(R+S) + q(1-\alpha) - (1-\alpha)\frac{dq}{dS}K'(q) \\
&= \frac{dq}{dS}(1-\alpha)[R + S - K'(q)] + q(1-\alpha) \\
&= \frac{dq}{dS}(1-\alpha)[\underbrace{R - \frac{1-\alpha}{\alpha}\beta S - K'(q)}_{=0} + \frac{1-\alpha}{\alpha}\beta S + S] + q(1-\alpha) \\
&= \underbrace{\frac{dq}{dS}}_{(-)}(1-\alpha)\underbrace{[\frac{1-\alpha}{\alpha}\beta S + S]}_{(+)} + \underbrace{q(1-\alpha)}_{(+)}
\end{aligned}$$

Note that for $S = 0$, the first derivative for U_{HC} is positive, whereas for S getting very large, the derivative gets negative. Hence, HC's payoff is maximized at $S > 0$.

Finally, note that

$$\frac{d(U_{MNE}+U_{HC})}{dS} = q(1-\alpha)(1-\beta) + \underbrace{\frac{dq}{dS}}_{(-)}(1-\alpha)\underbrace{[\frac{1-\alpha}{\alpha}\beta S + S]}_{(+)}$$

For $\beta \geq 1$, this derivative is negative for all S, whereas for $\beta < 1$ it is positive for $S = 0$ and it gets negative for large S. Hence welfare is maximized for $S > 0$.

Q.E.D.

Proof of Result 2

Using the results from above we can show that

$$\begin{aligned}\frac{dU_{MNE}}{d\alpha} &= qR + \frac{dq}{d\alpha}\alpha R + q\beta S - \frac{dq}{d\alpha}(1-\alpha)\beta S - K(q) - \alpha\frac{dq}{d\alpha}K'(q) \\ &= \frac{dq}{d\alpha}\underbrace{[\alpha R - (1-\alpha)\beta S - \alpha K'(q)]}_{=0} + q(R+\beta S) - K(q) > 0\end{aligned}$$

Furthermore,

$$\begin{aligned}\frac{dU_{HC}}{d\alpha} &= \frac{dq}{d\alpha}(1-\alpha)(R+S) - q(R+S) + K(q) - (1-\alpha)\frac{dq}{d\alpha}K'(q) \\ &= \frac{dq}{d\alpha}(1-\alpha)[\underbrace{R - \frac{1-\alpha}{\alpha}\beta S - K'(q)}_{=0} + \frac{1-\alpha}{\alpha}\beta S + S] - [q(R+S) - K(q)] \\ &= \underbrace{\frac{dq}{d\alpha}}_{(+)}(1-\alpha)[\frac{1-\alpha}{\alpha}\beta S + S] - \underbrace{[q(R+S) - K(q)]}_{(+)}\end{aligned}$$

Note that for $\alpha = 1$, this derivative is negative. For $\alpha \to 0$ it gets positive. Hence, HC's payoff is maximized at $\alpha < 1$.

Finally, note that

$$\frac{d(U_{MNE}+U_{HC})}{d\alpha} = q(\beta-1)S) + \underbrace{\frac{dq}{d\alpha}}_{(+)}\frac{(1-\alpha)}{\alpha}[(1-\alpha)\beta S + \alpha S]$$

This expression is positive for $\beta \geq 1$ and welfare is maximized for $\alpha < 1$ if $\beta < 1$.

Lemma A

For any $\alpha \in (0,1)$, HC's maximization problem has a unique interior solution

$T^*(\alpha) \in \left(-(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S, R - \frac{1-\alpha}{\alpha}\beta S\right)$.

Proof of Lemma A

We first show that HC's profit function is strictly concave in T. By the implicit function theorem, $\frac{dq^T(T)}{dT} = -\frac{1}{K''(q^T)} < 0$. Differentiating U_{HC}^T with respect to T we get

$$\begin{aligned}
\frac{dU_{HC}^T}{dT} &= \frac{dq^T(T)}{dT}\left[(1-\alpha)[\underbrace{R - T - K'(q^T)}_{=\frac{1-\alpha}{\alpha}\beta S \text{ by (7)}} + S] + T\right] + \alpha q^T(T) \\
&= -\frac{1}{K''(q^T)}\left[(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S + T\right] + \alpha q^T(T).
\end{aligned}$$

$$\frac{d^2U_{HC}^T}{dT^2} = -\frac{1}{K''}\left[\frac{K'''}{[K'']^2}[(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S + T] + 1 + \alpha\right] < 0.$$

Hence, the optimal $T^*(\alpha)$ must be unique. Furthermore, it is never optimal to choose $T \geq R - \frac{1-\alpha}{\alpha}\beta S$, because this would imply $q^T(T,\alpha) = 0$ and $U_{HC}^T = 0$, while a strictly positive payoff can be obtained by choosing $T < R - \frac{1-\alpha}{\alpha}\beta S$. Finally, it cannot be optimal to choose $T = \underline{T} \equiv -(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S$. To see this note that at $T = \underline{T}$ we have $q^T(T,\alpha) > 0$.Thus,

$$\frac{dU_{HC}^T}{dT}|_{T=\underline{T}} = \alpha q^T(T,\alpha) > 0.$$

Hence, if $\alpha > 0$, a strictly higher payoff can be obtained by choosing $T > \underline{T}$.

Q.E.D.

Proof of Lemma 1

By the implicit function theorem we can show that

$$\frac{dq^T}{dS} = -\frac{1}{K''}\left[\frac{1-\alpha}{\alpha}\beta + \frac{dT^*}{dS}\right].$$

Using again the implicit function theorem and taking account of the direct effect of an increase in S on q, i.e. $-\frac{1}{K''}\frac{1-\alpha}{\alpha}\beta$, we find that

$$\begin{aligned}
\frac{dT^*}{dS} &= -\frac{\frac{K'''}{[K'']^2}\frac{dq^T}{dS}[(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S+T^*]+\frac{dq^T}{dT}(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}+\alpha\frac{dq^T}{dS}}{\frac{K'''}{[K'']^2}\frac{dq^T}{dT}[(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S+T^*]+\frac{dq^T}{dT}+\alpha\frac{dq^T}{dT}} \\
&= -\left(\frac{1-\alpha}{\alpha}\beta\right)\underbrace{\frac{\frac{K'''}{[K'']^2}[(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S+T^*]+1+\frac{\alpha}{\beta}}{\frac{K'''}{[K'']^2}[(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S+T^*]+1+\alpha}}_{=A>0} \\
&= -\left(\frac{1-\alpha}{\alpha}\beta\right)A<0, \qquad \text{with } A \gtreqless 1 \text{ if } \beta \lesseqgtr 1.
\end{aligned}$$

Q.E.D.

Proof of Result 3

Using Lemma 1, we can show that

$$\frac{dq^T}{dS}=\frac{1}{K''}\frac{1-\alpha}{\alpha}\beta[A-1].$$

Differentiating U^T_{MNE} and U^T_{HC} with respect to S and re-arranging we get:

$$\begin{aligned}
\frac{dU^T_{MNE}}{dS} &= -q^T\alpha\frac{dT^*}{dS}-q^T(1-\alpha)\beta \\
&\quad \underbrace{+\frac{dq^T}{dS}\alpha\underbrace{[R-T^*-K'(q)]}_{=\frac{1-\alpha}{\alpha}\beta S\text{ by (7)}}-\frac{dq^T}{dS}(1-\alpha)\beta S}_{=0} \\
&= q^T(1-\alpha)\beta[A-1].
\end{aligned}$$

$$\begin{aligned}
\frac{dU^T_{HC}}{dS} &= \frac{dq^T}{dS}\left[(1-\alpha)[\underbrace{R-T^*-K'(q)}_{=\frac{1-\alpha}{\alpha}\beta S\text{ by (7)}}+S]+T^*\right]+q^T\left[(1-\alpha)+\alpha\frac{dT^*}{dS}\right] \\
&= \frac{dq^T}{dS}\left[(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S+T^*\right]+q^T(1-\alpha)[1-\beta A].
\end{aligned}$$

Note that for $\beta>1$ we get $\beta A>1$ and vice versa. Summarizing the effects:

(i) $\beta=1\Rightarrow A=1\Rightarrow \frac{dT^*}{dS}<0, \frac{dq^T}{dS}, \frac{dU^T_{MNE}}{dS}, \frac{dU^T_{HC}}{dS}=0, \Rightarrow \frac{d(U^T_{MNE}+U^T_{HC})}{dS}=0$

(ii) $\beta<1\Rightarrow A>1\Rightarrow \frac{dT^*}{dS}<0, \frac{dq^T}{dS}, \frac{dU^T_{MNE}}{dS}, \frac{dU^T_{HC}}{dS}>0, \Rightarrow \frac{d(U^T_{MNE}+U^T_{HC})}{dS}>0$

(iii) $\beta > 1 \Rightarrow A < 1 \Rightarrow \frac{dT^*}{dS} < 0, \frac{dq^T}{dS}, \frac{dU^T_{MNE}}{dS}, \frac{dU^T_{HC}}{dS} < 0, \Rightarrow \frac{d(U^T_{MNE}+U^T_{HC})}{dS} < 0$

For $\beta = 1$ we have $\frac{dq^T}{dS} = 0$. Thus, it follows from equation (7) that for $\beta = 1$ we must have $T^*(\alpha; S) = T^*(\alpha, 0) - \frac{1-\alpha}{\alpha}S$, where T^* characterizes the optimal choice of T for $S = 0$.

Q.E.D.

Proof of Lemma 2

By the implicit function theorem, it is straightforward to show that

$$\frac{dq^T}{d\alpha} = -\frac{1}{K''}\left[\frac{dT^*}{d\alpha} - \frac{1}{\alpha^2}\beta S\right].$$

Using again the implicit function theorem and taking account of the direct effect of an increase in α on $q^T(T,\alpha)$, i.e. $\frac{1}{K''}\frac{1}{\alpha^2}\beta S$, we can show that

$$\begin{aligned}\frac{dT^*}{d\alpha} &= \frac{1}{\alpha^2}\beta S \underbrace{\frac{\frac{K'''}{[K'']^2}[(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S + T^*] + 1 - \alpha^2 + \frac{\alpha^2}{\beta} + \alpha}{\frac{K'''}{[K'']^2}[(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S + T^*] + 1 + \alpha}}_{=B>0} \\ &+ \underbrace{\frac{q^S(T)}{\frac{K'''}{[K'']^3}[(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S + T^*] + (1+\alpha)\frac{1}{K''}}}_{=D>0} \\ &= \frac{1}{\alpha^2}\beta S B + D > 0, \text{ with} B \gtreqless 1 \text{ for } \beta \lesseqgtr 1.\end{aligned}$$

Q.E.D.

Proof of Result 4

Differentiating U^T_{HC} with respect to α and re-arranging we get:

$$\begin{aligned}\frac{dU^T_{HC}}{d\alpha} &= \frac{dq^T}{d\alpha}\left[(1-\alpha)[\underbrace{R - T^* - K'(q^T)}_{=\frac{1-\alpha}{\alpha}\beta S \text{ by (7)}} + S] + T^*\right] + \alpha q^T \frac{dT^*}{d\alpha} \\ &+ K(q^T) - q^T[R - T^* + S] \\ &= -\frac{1}{K''}\left[\frac{dT^*}{d\alpha} - \frac{1}{\alpha^2}\beta S\right]\left[\underbrace{(1-\alpha)\frac{\beta - \alpha\beta + \alpha}{\alpha}S + T^*}_{=K''\alpha q^T \text{ by (8)}}\right] + \alpha q^T \frac{dT^*}{d\alpha}\end{aligned}$$

$$
\begin{aligned}
& + K(q^T) - q^T[R - T^* + S] \\
= \; & \underbrace{K(q^T) - q^T\left[R - T^* + S\right]}_{<0} + \underbrace{q^T \frac{1}{\alpha}\beta S}_{>0}. \qquad (A1)
\end{aligned}
$$

A marginal increase of α reduces HC's share of total surplus, $q^T[R-T^*]-K(q^T)$, and reduces the received spillover, $q^T S$. On the other hand, a marginal increase of α induces HC to increase total taxation by $\frac{dT^*}{d\alpha}$ and it induces MNE to change investment by $\frac{dq^T}{d\alpha}$. Both effects sum up to $q^T \frac{1}{\alpha}\beta S$, which is basically the direct effect of an increase in α on the investment q^T. This effect may dominate and thus HC may prefer to increase α, if β is sufficiently large. Note that (A1) can be positive only if $\beta > 1$. To see this, note further that MNE will choose $q^T > 0$ only if $U^T_{MNE} > 0$, i.e.

$$
q^T \alpha[R - T^*] - q^T(1-\alpha)\beta S - \alpha K(q^T) - I > 0.
$$

Condition (A1) is positive if, after re-arranging, we have

$$
q^T \alpha\left[R - T^*\right] - q^T \beta S + \alpha q^T S - \alpha K(q^T) < 0.
$$

Both conditions can be fulfilled simultaneously only if $\beta > 1$.

Differentiating U^T_{MNE} with respect to α and re-arranging we get:

$$
\begin{aligned}
\frac{dU^T_{MNE}}{d\alpha} &= q^T[R - T^* + \beta S] - K(q^T) - q^T \alpha \frac{dT^*}{d\alpha} \\
& \quad \underbrace{+ \frac{dq^T}{d\alpha}\alpha \underbrace{\left[R - T^* - K'(q^T)\right]}_{=\frac{1-\alpha}{\alpha}\beta S \text{ by } (7)} - \frac{dq^T}{d\alpha}(1-\alpha)\beta S}_{=0} \\
&= \underbrace{q^T[R - T^* + \beta S] - K(q^T)}_{>0} - \underbrace{\alpha q^T \frac{dT^*}{d\alpha}}_{>0}. \qquad (A2)
\end{aligned}
$$

Thus, the impact of α on MNE's payoff may be ambiguous. A marginal increase of α increases MNE's share of the total net payoff, $q^T[R - T^*] - K(q^T)$, and reduces the loss due to the spillover, $q^T \beta S$. On the other hand, a marginal increase of α induces HC to increase total taxes by $\frac{dT^*}{d\alpha}$, of which MNE has to pay the share α in case of a successful project, which happens with probability q^T. If α is close enough to 0, the second effect vanishes and MNE always prefers to increase α. However, if α is sufficiently large, the second effect may dominate. The effect of a change of α on total surplus is given by

$$\begin{aligned} \frac{d(U^T_{MNE}+U^T_{HC})}{d\alpha} &= q^T\frac{1}{\alpha}\beta S - q^T\alpha\frac{dT^*}{d\alpha} + q^T(\beta-1)S \\ &= \underbrace{q^T\frac{1}{\alpha}\beta S[1-B]}_{\gtrless 0 \text{ for } \beta \gtrless 1} - \underbrace{\alpha q^T D}_{>0} + \underbrace{q^T(\beta-1)S}_{\gtrless 0 \text{ for } \beta\gtrless 1}. \end{aligned} \qquad (A3)$$

By proof of Lemma 1 and Result 3 we know that for $\beta = 1$, $T^*(\alpha, S) = T^*(\alpha, 0) - \frac{1-\alpha}{\alpha}S$ and thus $q^T(\alpha; S) = q^*(\alpha, 0)$. Hence, equations (A1), (A2), and (A3), and therefore the effects of a decrease in α are the same for $\beta = 1$ and for $S = 0$.

Summarizing the effects:

(i) $\beta = 1 \Rightarrow \frac{dT^*}{d\alpha} > 0, \frac{dq^T}{d\alpha} < 0, \frac{dU^T_{MNE}}{d\alpha} \gtrless 0, \frac{dU^T_{HC}}{d\alpha} < 0, \frac{d(U^T_{MNE}+U^T_{HC})}{d\alpha} < 0.$

(ii) $\beta < 1 \Rightarrow \frac{dT^*}{d\alpha} > 0, \frac{dq^T}{d\alpha} < 0, \frac{dU^T_{MNE}}{d\alpha} \gtrless 0, \frac{dU^T_{HC}}{d\alpha} < 0, \frac{d(U^T_{MNE}+U^T_{HC})}{d\alpha} < 0.$

(iii) $\beta > 1 \Rightarrow \frac{dT^*}{d\alpha} > 0, \frac{dq^T}{d\alpha} \gtrless 0, \frac{dU^T_{MNE}}{d\alpha} \gtrless 0, \frac{dU^T_{HC}}{d\alpha} \gtrless 0, \frac{d(U^T_{MNE}+U^T_{HC})}{d\alpha} \gtrless 0.$

We prove by example that there indeed exist cases with the properties described in the Result. Consider the following cost function:

$$K(q) = \frac{1}{1-q} - q.$$

For $\alpha = 0.98$, $R = 40$, and $S = 3$ the following results are obtained for different values of β:

	$\frac{dU^T_{MNE}}{d\alpha}$	$\frac{dU^T_{HC}}{d\alpha}$	$\frac{d(U^T_{MNE}+U^T_{HC})}{d\alpha}$	q^T	T^*	U^T_{MNE}	U^T_{HC}
$\beta = 0.3$	**-0.25**	-4.11	-4.36	0.65866	32.40	2.669	21.434
$\beta = 1$	**-0.07**	-2.72	-2.79	0.65855	32.36	2.665	21.406
$\beta = 1.2$	**-0.02**	-2.32	-2.34	0.65852	32.35	2.664	21.398
$\beta = 3$	0.43	**1.23**	**1.67**	0.65823	32.25	2.655	21.327

Thus, for large values of α, there exist cases where MNE's payoff increases as α decreases. This result can be obtained independently of the efficiency of a spillover β. For $\beta > 1$ there exist cases where HC's payoff and the efficiency of the project increase as α increases. This is the case for $\beta = 3$ in the example. For $\alpha = 1$ and $R = 40$ we get:

	$\frac{dU^T_{MNE}}{d\alpha}$	$\frac{dU^T_{HC}}{d\alpha}$	$\frac{d(U^T_{MNE}+U^T_{HC})}{d\alpha}$	q^T	T^*	U^T_{MNE}	U^T_{HC}
$\alpha = 1$	-	-	-	0.65838	32.51	2.664	21.352

Hence, in some cases HC benefits and the efficiency of the project is maximized if ownership is not shared. The intuition for this is easy to see for the extreme example of β being so large that the optimal q is chosen equal to zero for any $\alpha < 1$. In this case, HC's payoff is maximized by choosing $\alpha = 1$. This induces MNE to choose a positive q and hence allows HC to enjoy a positive payoff through taxation.

Q.E.D.

Lemma B

For any $\alpha \in (0, 1)$, HC's payoff is maximized at $M^*(\alpha)$, with

(a) $M^*(\alpha) \in (0, \infty)$, if $R > \frac{1-\alpha}{\alpha}\beta S$ and $\frac{d^2U^M_{HC}}{dM^2}|_{M=M^*} < 0$, or

(b) $M^*(\alpha) \in (\frac{1-\alpha}{\alpha}\beta S - R, \infty)$, if $R \leq \frac{1-\alpha}{\alpha}\beta S$, $\frac{d^2U^M_{HC}}{dM^2}|_{M=M^*} < 0$, and $U^M_{HC}(M^*) > 0$, or

(c) $M^* = 0$, if $R \leq \frac{1-\alpha}{\alpha}\beta S$ otherwise.

Proof of Lemma B

The optimal level of investment q is characterized by the following first order condition:

$$K'(q^M) = R + M - \frac{1-\alpha}{\alpha}\beta S. \qquad (A4)$$

By the implicit function theorem, $\frac{dq^M(M)}{dM} = \frac{1}{K''(q^M)} > 0$. Differentiating U^M_{HC} with respect to M we get

$$\begin{aligned} \frac{dU^M_{HC}}{dM} &= \frac{dq^M}{dM}(1-\alpha)\left[\underbrace{R + M - K'(q^M)}_{=\frac{1-\alpha}{\alpha}\beta S \text{ by (9)}} + S\right] + q^M(1-\alpha) - C'(M) \\ &= \frac{1}{K''(q^M)}(1-\alpha)\frac{\beta - \alpha\beta + \alpha}{\alpha}S + q^M(M)(1-\alpha) - C'(M). \end{aligned}$$

$$\frac{d^2U_{HC}^M}{dM^2} = -\frac{1}{K''}\left[\frac{K'''}{[K'']^2}(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S-(1-\alpha)\right]-C''(M).$$

Hence, HC's payoff is maximized at $M^*(\alpha)$, if $\frac{d^2U_{HC}^M}{dM^2}|_{M=M^*} < 0$ and if moreover $U_{HC}^M(M^*) > 0$. Given the assumptions on $C(M)$ there must exist an upper bound for M^*.

If $\alpha < 1$ and $R > \frac{1-\alpha}{\alpha}\beta S$, it is never optimal to choose $M = 0$. To see this note that in this case $q^M > 0$ and thus

$$\left.\frac{dU_{HC}^M}{dM}\right|_{M=0} = \frac{1}{K''(q^M)}(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S + q^M(M)(1-\alpha) > 0.$$

Hence, if $\alpha \in (0,1)$, a strictly higher payoff can be obtained by choosing $M > 0$.

If $\alpha < 1$ and $R \leq \frac{1-\alpha}{\alpha}\beta S$, it follows from equation (9) that $q = 0$ for all $M \leq \frac{1-\alpha}{\alpha}\beta S - R$. Hence, HC chooses $M^* \in (\frac{1-\alpha}{\alpha}\beta S - R, \infty)$ if $U_{HC}^M(M^*) > 0$ and $M^* = 0$ otherwise.

Q.E.D.

<u>Proof of Lemma 3</u>

By the implicit function theorem we can show that

$$\frac{dq^M}{dS} = -\frac{1}{K''}\left[\frac{1-\alpha}{\alpha}\beta - \frac{dM^*}{dS}\right].$$

Using again the implicit function theorem and taking account of the direct effect of an increase in S on $q^M(M,\alpha)$, i.e. $-\frac{1}{K''}\frac{1-\alpha}{\alpha}\beta S$, we find that

$$\begin{aligned}\frac{dM^*}{dS} &= -\frac{-\frac{K'''}{[K'']^2}\frac{dq^M}{dS}(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S + \frac{dq^M}{dM}(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha} + \frac{dq^M}{dS}(1-\alpha)}{-\frac{K'''}{[K'']^2}\frac{dq^M}{dM}(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S + \frac{dq^M}{dM}(1-\alpha) - C''} \\ &= \frac{1-\alpha}{\alpha}\beta\,\frac{\frac{K'''}{[K'']^2}(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S + \frac{\alpha}{\beta}}{\underbrace{\frac{K'''}{[K'']^2}(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S - (1-\alpha) + K''C''}_{=E>0}} > 0.\end{aligned}$$

The last inequality follows from the fact that the denominator has to be positive by Lemma B if HC's payoff is maximized at $M^*(\alpha)$.

Q.E.D.

Proof of Result 5

From Lemma 3 it follows

$$\frac{dq^M}{dS} = \frac{1}{K''}\frac{1-\alpha}{\alpha}\beta[E-1] \gtreqless 0,$$

where E is defined above in the Proof of Lemma 3. Note that $E > 1$ if $\frac{\alpha}{\beta} > -(1-\alpha) + K''C''$ and vice versa. It is straightforward to see that $E > 1$ is compatible with both $\beta > 1$ and $\beta < 1$, and vice versa for $E < 1$.

Differentiating U^M_{MNE} and U^M_{HC} with respect to S and re-arranging we get:

$$\begin{aligned}
\frac{dU^M_{MNE}}{dS} &= \underbrace{\frac{dq^M}{dS}\alpha\underbrace{[R+M-K'(q^M)]}_{=\frac{1-\alpha}{\alpha}\beta S \text{ by (A4)}} - \frac{dq^M}{dS}(1-\alpha)\beta S}_{=0} \\
&\quad + q^M\alpha\frac{dM^*}{dS} - q^M(1-\alpha)\beta \\
&= q^M(1-\alpha)\beta[E-1] \gtreqless 0.
\end{aligned}$$

$$\begin{aligned}
\frac{dU^M_{HC}}{dS} &= \frac{dq^M}{dS}(1-\alpha)\left[\underbrace{R+M-K'(q^M)}_{=\frac{1-\alpha}{\alpha}\beta S \text{ by (A4)}} + S\right] + q^M(1-\alpha)\frac{dM^*}{dS} \\
&\quad + q^M(1-\alpha) - C'(M)\frac{dM^*}{dS} \\
&= -\frac{1}{K''}\left[\frac{1-\alpha}{\alpha}\beta - \frac{dM^*}{dS}\right](1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S + q^M(1-\alpha)\frac{dM^*}{dS} \\
&\quad + q^M(1-\alpha) - C'(M)\frac{dM^*}{dS} \\
&= -\frac{1}{K''}\frac{(1-\alpha)^2}{\alpha}\left(\frac{\beta-\alpha\beta+\alpha}{\alpha}\right)\beta S + q^M(1-\alpha) \gtreqless 0.
\end{aligned}$$

The effect of a change in S on total surplus is given by

$$\begin{aligned}
\frac{d(U^M_{MNE}+U^M_{HC})}{dS} &= q^M(1-\alpha)\beta[E-1] - \frac{1}{K''}\frac{(1-\alpha)^2}{\alpha}\frac{\beta-\alpha\beta+\alpha}{\alpha}\beta S \\
&\quad + q^M(1-\alpha) \gtreqless 0.
\end{aligned}$$

Q.E.D.

Proof of Lemma 4

By the implicit function theorem we can show that

$$\frac{dq^M}{d\alpha} = \frac{1}{K''}\left[\frac{dM^*}{d\alpha} + \frac{1}{\alpha^2}\beta S\right].$$

Using again the implicit function theorem and taking account of the direct effect of an increase in α on $q^M(M, \alpha)$, i.e. $\frac{1}{K''}\frac{1}{\alpha^2}\beta S$, we can show that

$$\begin{aligned} \frac{dM^*}{d\alpha} &= -\frac{1}{\alpha^2}\beta S \frac{\frac{K'''}{[K'']^2}[(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S] + \alpha - \alpha^2 + \frac{\alpha^2}{\beta}}{\frac{K'''}{[K'']^2}[(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S] - (1-\alpha) + K''C''} \\ &\quad - \frac{q^M(M)}{\frac{K'''}{[K'']^3}[(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha}S] - (1-\alpha)\frac{1}{K''} + C''} \qquad < 0. \end{aligned}$$

The last inequality follows from the fact that the denominator has to be positive by Lemma B if HC's payoff is maximized at $M^*(\alpha)$.

Q.E.D.

Proof of Result 6

Differentiating U^M_{HC} and U^M_{MNE} with respect to α and re-arranging we get:

$$\begin{aligned} \frac{dU^M_{HC}}{d\alpha} &= \frac{dq^M}{d\alpha}(1-\alpha)\left[\underbrace{R + M - K'(q^M)}_{=\frac{1-\alpha}{\alpha}\beta S \text{ by } (9)} + S\right] + q^M(1-\alpha)\frac{dM^*}{d\alpha} \\ &\quad - C'(M)\frac{dM^*}{d\alpha} + K(q^M) - q^M[R + M^* + S] \\ &= \frac{1}{K''}\left[\frac{dM^*}{d\alpha} + \frac{1}{\alpha^2}\beta S\right](1-\alpha)\frac{\beta - \alpha\beta + \alpha}{\alpha}S \\ &\quad + \frac{dM^*}{d\alpha}\left[q^M(1-\alpha) - C'(M)\right] + K(q^M) - q^M[R + M^* + S] \\ &= \underbrace{K(q^M) - q^M[R + M^* + S]}_{<0} + \underbrace{\frac{1}{K''}(1-\alpha)\frac{\beta - \alpha\beta + \alpha}{\alpha^3}\beta S^2}_{>0}. \end{aligned} \qquad (A5)$$

Thus, the impact of α on HC's payoff is, independently of the efficiency of the spillover β, ambiguous. A marginal increase of α reduces HC's share of total surplus, $K(q^M) - q^M[R + M^*]$ and reduces the received spillover $q^M S$. On the other hand, a marginal increase of α induces HC to reduce its investment in infrastructure by $\frac{dM^*}{d\alpha}$ and it induces MNE to change investment by $\frac{dq^M}{d\alpha}$. Both effects sum up to the second expression in (A5). This effect may dominate depending on the exact nature of investment costs.

$$\begin{aligned}\frac{dU^M_{MNE}}{d\alpha} &= q^M[R+M^*+\beta S] - K(q^M) + \alpha q^M \frac{dM^*}{d\alpha} \\ &\quad \underbrace{+ \frac{dq^M}{d\alpha}\alpha\underbrace{[R+M-K'(q^M)]}_{=\frac{1-\alpha}{\alpha}\beta S \text{ by (A4)}} - \frac{dq^M}{d\alpha}(1-\alpha)\beta S}_{=0} \\ &= \underbrace{q^M[R+M^*+\beta S] - K(q^M)}_{>0} + \underbrace{\alpha q^M \frac{dM^*}{d\alpha}}_{<0}.\end{aligned}$$

The impact of α on MNE's payoff may also be ambiguous. A marginal increase of α increases MNE's share of total net payoff, $q^M[R+M^*] - K(q^M)$ and reduces the loss due to the spillover by $q^M\beta S$. On the other hand, a marginal increase of α induces HC to reduce its investment in infrastructure by $\frac{dM^*}{d\alpha}$, of which MNE enjoys the share α in case of a successful project, which happens with probability q^M. If α is close to 0, the second effect vanishes and MNE always prefers to increase α. However, if α is sufficiently large, the second effect may dominate. The effect of a change in α on total surplus is given by

$$\frac{d(U^M_{MNE}+U^M_{HC})}{d\alpha} = \underbrace{q^M(\beta-1)S}_{\gtreqless 0 \text{ for } \beta \gtreqless 1} + \underbrace{\frac{1}{K''}(1-\alpha)\frac{\beta-\alpha\beta+\alpha}{\alpha^3}\beta S^2}_{>0} + \underbrace{\alpha q^M \frac{dM^*}{d\alpha}}_{<0}.$$

Summarizing the effects:

(i) $S = 0 \Rightarrow \frac{dM^*}{d\alpha} < 0, \frac{dq^M}{d\alpha} < 0, \frac{dU^M_{MNE}}{d\alpha} \gtreqless 0, \frac{dU^M_{HC}}{d\alpha} < 0, \frac{d(U^M_{MNE}+U^M_{HC})}{d\alpha} < 0.$

(ii) $S > 0 \Rightarrow \frac{dM^*}{d\alpha} < 0, \frac{dq^M}{d\alpha} \gtreqless 0, \frac{dU^M_{MNE}}{d\alpha} \gtreqless 0, \frac{dU^M_{HC}}{d\alpha} \gtreqless 0, \frac{d(U^M_{MNE}+U^M_{HC})}{d\alpha} \gtreqless 0.$

We prove by example that there indeed exist cases with the properties described in the Result. Consider the following cost functions:

$$K(q) = \frac{1}{3}q^3 \text{ and } C(M) = M^2.$$

For $\alpha = 0.98$, $R = 0.1$, and $S = 20$ the following results are obtained for different values of β:

	$\frac{dU^M_{MNE}}{d\alpha}$	$\frac{dU^M_{HC}}{d\alpha}$	$\frac{d(U^M_{MNE}+U^M_{HC})}{d\alpha}$	q^M	M^*	U^M_{MNE}	U^M_{HC}
$\beta = 0.4$	**-0.92**	-4.62	-5.54	0.423	0.242	0.05	0.11
$\beta = 0.8$	0.03	**4.32**	**4.35**	0.315	0.326	0.02	0.02
$\beta = 0.9$	-	-	-	**0**	**0**	0	0

Thus, for large values of α, there exist cases where MNE's payoff increases as α decreases. Moreover, there exist cases where HC's payoff and the efficiency of the project increase as α increases. As the example highlights this can be the case even for an efficient spillover. For $\beta = 0.9$ sharing of ownership with $\alpha = 0.98$ results in no investment by both parties. However, for $\alpha = 0.99$, $R = 0.1$, and $S = 20$ we get:

	$\frac{dU^M_{MNE}}{d\alpha}$	$\frac{dU^M_{HC}}{d\alpha}$	$\frac{d(U^M_{MNE}+U^M_{HC})}{d\alpha}$	q^M	M^*	U^M_{MNE}	U^M_{HC}
$\beta = 0.9$	0.19	0.18	0.37	0.298	0.171	0.02	0.03

Q.E.D.