Who is Afraid of Political Risk?  
Multinational Firms and their Choice of Capital Structure

Iris Kesternich\textsuperscript{a} and Monika Schnitzer\textsuperscript{b}

\textsuperscript{a} University of Munich, Department of Economics, Akademiestr. 1, D-80799 Munich, Germany. Phone: +49 89 2180 3232, Fax: +49 89 2180 2767, e-mail: Iris.Kesternich@lrz.uni-muenchen.de

\textsuperscript{b} Corresponding author: University of Munich, Department of Economics, Akademiestr. 1, D-80799 Munich, Germany, and Centre for Economic Policy Research. Phone: +49 89 2180 2217, Fax: +49 89 2180 2767, e-mail: schnitzer@lrz.uni-muenchen.de

Forthcoming in Journal of International Economics  
Accepted Version: July 2010

Mathematical Appendix

\textbf{Proof of Result 1}

Recall that the interest rate is implicitly determined by the following break even condition for the bank:

\[ \int_{(1+r)D}^{\bar{R}} (1+r)D \frac{1}{\bar{R}} d\bar{R} + \int_{0}^{(1+r)D} sR \frac{1}{\bar{R}} d\bar{R} = D \]  

(1)

Solving and rearranging yields

\[ \frac{1}{\bar{R}} \left[ (1+r)D\bar{R} - \frac{2-s}{2} (1+r)^2 D^2 \right] = D \]  

(2)

\[ \frac{1}{\bar{R}}^2 (1+r)^2 D^2 = \frac{(1+r)D}{2-s} - \frac{D}{2-s} \]  

(3)

\[ (2-s)(1+r)^2 D = 2r \bar{R} \]  

(4)

From the implicit function

\[ 2r \bar{R} - (2-s)(1+r)^2 D = 0 \]  

(5)
we can derive how the interest rate reacts for a change in the debt level $D$, using the implicit function theorem.

$$\frac{dr}{dD} = -\frac{-(2-s)(1+r)^2}{2[R-(2-s)(1+r)D]}$$

(6)

Using (5), this simplifies to

$$\frac{dr}{dD} = \frac{r(1+r)}{(1-r)D}$$

(7)

Furthermore, we can use (5) to derive how the interest rate reacts to a change in $\bar{R}$, again using the implicit function theorem.

$$\frac{dr}{d\bar{R}} = -\frac{-2r}{2[R-(2-s)(1+r)D]}$$

(8)

Using (5), this simplifies to

$$\frac{dr}{d\bar{R}} = -\frac{r(1+r)}{(1-r)\bar{R}}$$

(9)

The investor maximizes

$$U(D, \alpha) = V(D, \alpha) + B(\alpha) - K(\alpha)$$

(10)

$$= \int_{\bar{R}(\alpha)}^{1/(1+r)} (1-t)[R-(1+r)D]\frac{1}{R(\alpha)}dR + D - I + B(\alpha) - K(\alpha)$$

(11)

$$= \frac{1-t}{R(\alpha)} \left[ \frac{1}{2} \bar{R}^2(\alpha) - (1+r)D\bar{R}(\alpha) + \frac{1}{2}(1+r)^2D^2 \right] + D - I + B(\alpha) - K(\alpha)$$

Using equation (1) we can rewrite the payoff function in the following way

$$U(D, \alpha) = (1-t) \left[ \frac{1}{2} \bar{R}(\alpha) - \frac{(1-s)(1+r)D}{2-s} \right] + \frac{1-s+t}{2-s}D - I + B(\alpha) - K(\alpha).$$

(12)

The investor’s maximization problem is characterized by the following two first order conditions.

$$\frac{dU}{d\alpha} = (1-t) \left[ \frac{1}{2} \frac{d\bar{R}}{d\alpha} - \frac{1-s}{2-s} D \frac{dr}{d\bar{R}} \right] + B' - K' = 0$$

(13)

$$\frac{dU}{dD} = -(1-t) \frac{1-s}{2-s} \left[ (1+r) + \frac{dr}{dD} D \right] + \frac{1-s+t}{2-s} = 0$$

(14)

Using (7), we can rewrite the first order condition for the optimal $D$ in the following way

$$\frac{dU}{dD} = -(1-t) \frac{1-s}{2-s} \left[ 1 + r + \frac{r(1+r)}{(1-r)D} \right] + \frac{1-s+t}{2-s} = 0$$

(15)
Rearranging yields
\[
\frac{1 - s + t}{2 - s} = (1 - t) \frac{1 - s + r}{2 - s - r}
\] (16)

We can use this condition to solve for \( r \) and \( 1 + r \):
\[
r = \frac{(2 - s)t}{2(1 - s) + st} \quad 1 + r = \frac{2(1 - s + t)}{2(1 - s) + st}
\] (17)

Inserting this in (5), we can solve for
\[
D^* = \frac{\bar{R} \cdot 2(1 - s) + st}{2(1 - s + t)^2}
\] (18)

Using the solutions for \( r \) and \( D \), we can finally determine the investor’s payoff as a function of \( \alpha \)
\[
U = \frac{\bar{R}(\alpha) \cdot 1 - s + st}{2 - 1 - s + t} - I + B(\alpha) - K(\alpha)
\] (19)

Q.E.D

**Proof of Result 2**

To see how the optimal debt level reacts to changes in \( t \), consider the optimal debt level as characterized by (18):
\[
\frac{dD^*}{dt} = \frac{\bar{R}}{2} \frac{2(1 - s) + st + t(1 - s + t)^2s - (2(1 - s) + st)2(1 - s + t)}{(1 - s + t)^4}
\] (20)

\[
= \frac{\bar{R}(1 - s)^2(1 - t)}{(1 - s + t)^3} > 0
\] (21)

To determine how the ownership ratio \( \alpha \) reacts to changes in \( t \), we rewrite the first order condition (13) that implicitly defines \( \alpha^* \), using (9) and the solution to \( D^* \) and \( r \). After some simplification we obtain
\[
(1 - t) \left[ 1 + \frac{t^2}{(1 - s + t)(1 - t)} \right] \frac{1}{2} \frac{dR}{d\alpha} + B' - K' = 0
\] (22)

\[
\left[ \frac{1 - s + st}{1 - s + t} \right] \frac{1}{2} \frac{dR}{d\alpha} + B' - K' = 0
\] (23)

From this we find, using the implicit function theorem
\[
\frac{d\alpha^*}{dt} = -\frac{\left[ \frac{(1 - s + t)s - (1 - s + st)}{(1 - s + t)^2} \right] \frac{dR}{d\alpha} }{\left[ \frac{1 - s + st}{1 - s + t} \right] \frac{1}{2} \frac{d^2R}{d\alpha^2} + B'' - K''}
\] (24)
\[
= \frac{\left(\frac{(1-s)^2}{1-s+t}\right) \frac{1}{2} \frac{dR}{d\alpha}}{\frac{1-s+st}{1-s+t}} + \frac{\frac{1}{2} \frac{d^2 R}{d\alpha^2}}{+} + \frac{B'' - K''}{+}
\]

(26)

Note that the sign of \(\frac{d\alpha}{dt}\) depends on the sign of \(\frac{d\bar{R}}{d\alpha}\). Using equation (23), we find that \(R' > 0\) if \(B' < K'\) in the relevant parameter range, and hence \(\frac{d\alpha}{dt} < 0\) if \(B' < K'\).

Q.E.D.

**Proof of Result 3**

Consider first the case of expropriation. Recall that the interest rate is implicitly determined by the following break even condition for the bank:

\[
(1 - \pi_1) \left[ \int_0^R (1 + r)D \frac{1}{R} dR + \int_0^{(1+r)D} sR \frac{1}{R} dR \right] = D
\]

(27)

Solving and rearranging yields

\[
\frac{1 - \pi_1}{R} \left[ (1 + r)D\bar{R} - \frac{2-s}{2}(1 + r)^2 D^2 \right] = D
\]

(28)

\[
\frac{1 - \pi_1}{R} \frac{1}{2}(1 + r)^2 D^2 = \frac{(1 - \pi_1)(1 + r)D}{2 - s} - \frac{D}{2 - s}
\]

(29)

\[
(1 - \pi_1)(2 - s)(1 + r)^2 D = 2(1 - \pi_1)(1 + r)\bar{R} - 2\bar{R}
\]

(30)

From the implicit function

\[
2[(1 - \pi_1)(1 + r) - 1]\bar{R} - (1 - \pi_1)(2 - s)(1 + r)^2 D = 0
\]

(31)

we can derive how the interest rate reacts to a change in the debt level \(D\)

\[
\frac{dr}{dD} = \frac{-(1 - \pi_1)(2-s)(1+r)^2}{2[(1 - \pi_1)R - (1 - \pi_1)(2-s)(1+r)D]}
\]

(32)

Using (31), this simplifies to

\[
\frac{dr}{dD} = \frac{(1 + r)[(1 - \pi_1)(1 + r) - 1]}{[2 - (1 - \pi_1)(1-r)]D}
\]

(33)

Furthermore, we can derive how the interest rate reacts to a change in \(\bar{R}\)

\[
\frac{dr}{d\bar{R}} = \frac{2[(1 - \pi_1)(1+r) - 1]}{2(1 - \pi_1)\bar{R} - 2(1 - \pi_1)(2-s)(1+r)D}
\]

(34)
Using again (31), this simplifies to

\[
\frac{dr}{R} = -\frac{2[(1 - \pi_1)(1 + r) - 1](1 + r)}{2 - (1 - \pi_1)(1 + r)R} > 0
\]  

(35)

The investor maximizes

\[
U_1(D, \alpha) = (1 - \pi_1) \int_{(1 + r)D}^{R(\alpha)} (1 - t)[R - (1 + r)D] dR + D - I + (1 - \pi_1)B(\alpha) - K(\alpha)
\]

\[
= \frac{(1 - \pi_1)(1 - t)}{R(\alpha)} \left[ \frac{1}{2} \bar{R}^2(\alpha) - (1 + r)D\bar{R}(\alpha) + \frac{1}{2}(1 + r)^2D^2 \right] + D - I + (1 - \pi_1)B(\alpha) - K(\alpha)
\]

(36)

Using equation (27) we can rewrite the payoff function in the following way

\[
U_1(D, \alpha) = (1 - \pi_1)(1 - t) \left[ \frac{1}{2} \bar{R}(\alpha) - \frac{(1 - s)(1 + r)}{2 - s} D \right] + \frac{1 - s + t}{2 - s} D - I + (1 - \pi_1)B(\alpha) - K(\alpha).
\]  

(37)

The investor’s maximization problem is characterized by the following two first order conditions.

\[
\frac{dU_1}{d\alpha} = (1 - \pi_1)(1 - t) \left[ \frac{1}{2} \frac{d\bar{R}}{d\alpha} - \frac{1 - s}{2 - s} D \frac{d\bar{R}}{d\alpha} \right] + (1 - \pi_1)B' - K' = 0
\]  

(38)

\[
\frac{dU_1}{dD} = -(1 - \pi_1)(1 - t) \frac{1 - s}{2 - s} \left[ (1 + r) + \frac{dr}{dD} D \right] + \frac{1 - s + t}{2 - s} = 0
\]  

(39)

Using (33), we can rewrite the first order condition for the optimal \(D\) in the following way

\[
\frac{dU_1}{dD} = -(1 - \pi)(1 - t) \frac{1 - s}{2 - s} \left[ 1 + r + \frac{(1 + r)[(1 - \pi_1)(1 + r) - 1]}{2 - (1 - \pi_1)(1 + r)D} D \right] + \frac{1 - s + t}{2 - s} = 0
\]  

(40)

Rearranging yields

\[
\frac{1 - s + t}{2 - s} = (1 - \pi)(1 - t) \frac{1 - s}{2 - s} \frac{1 + r}{2 - (1 - \pi_1)(1 + r)}
\]

(41)

We can use this condition to solve for \(r\) and \(1 + r\):

\[
r_1 = \frac{(2 - s)t}{(1 - \pi_1)[2(1 - s) + st]} \quad 1 + r_1 = \frac{2(1 - s + t)}{(1 - \pi_1)[2(1 - s) + st]}
\]  

(42)

Inserting this in (31), we can solve for

\[
D_1^* = (1 - \pi_1) \frac{\bar{R}t}{2} \frac{2(1 - s) + st}{(1 - s + t)^2}
\]

(43)
Using the solutions for \( r \) and \( D_1 \), we can finally determine the investor’s payoff

\[
U_1 = (1 - \pi_1) \left[ \frac{\bar{R}}{2} \frac{1 - s + s \bar{t}}{1 - s + t} + B(\alpha) \right] - K(\alpha) - I
\]

(44)

**Creeping expropriation**

Consider now the case of creeping expropriation. Recall that the interest rate is implicitly determined by the following break even condition for the bank:

\[
\left[ \int_{(1+r)D}^{(1+r)\bar{R}} (1+r)D \frac{1}{R} dR + \int_{0}^{(1+r)\bar{R}} (1-s)R \frac{1}{R} dR \right] = D
\]

(45)

Solving and rearranging yields

\[
\frac{1}{(1-\pi_2)\bar{R}} \left[ (1+r)D(1-\pi_2)\bar{R} - \frac{2-s}{2}(1+r)^2D^2 \right] = D
\]

(46)

\[
\frac{1}{(1-\pi_2)\bar{R}} \frac{1}{2}(1+r)^2D^2 = \frac{(1+r)D}{2-s} - \frac{D}{2-s}
\]

(47)

\[
(2-s)(1+r)^2D = 2(1-\pi_2)(1+r)\bar{R} - 2(1-\pi_2)\bar{R}
\]

\[
(2-s)(1+r)^2D = 2r(1-\pi_2)\bar{R}
\]

(48)

From the implicit function

\[
2r(1-\pi_2)\bar{R} - (2-s)(1+r)^2D = 0
\]

(49)

we can derive how the interest rate reacts for a change in the debt level \( D \)

\[
\frac{dr}{dD} = -\frac{-(2-s)(1+r)^2}{2[(1-\pi_2)\bar{R} - (2-s)(1+r)D]}
\]

(50)

Using (49), this simplifies to

\[
\frac{dr}{dD} = \frac{r(1+r)}{(1-r)\bar{R}}
\]

(51)

Furthermore, we can derive how the interest rate reacts to a change in \( \bar{R} \)

\[
\frac{dr}{d\bar{R}} = -\frac{2r(1-\pi_2)}{2(1-\pi_2)\bar{R} - 2(2-s)(1+r)D}
\]

(52)

Using again (49), this simplifies to

\[
\frac{dr}{\bar{R}} = -\frac{r(1+r)}{(1+r)\bar{R}} > 0
\]

(53)
The investor maximizes

\[ U_2(D, \alpha) = \int_{(1+\pi_2)D}^{R(\alpha)} (1 - \pi_2)R - (1 + r)D \frac{1}{R(\alpha)} dR + D - I + (1 - \pi_2)B(\alpha) - K(\alpha) \]

\[ = \frac{(1 - t)}{(1 - \pi_2)R(\alpha)} \left\{ \frac{1}{2} (1 - \pi_2)^2 \tilde{R}(\alpha) - (1 + r)D(1 - \pi_2)\tilde{R}(\alpha) + \frac{1}{2} (1 + r)^2 D^2 \right\} \]

\[ + D - I + (1 - \pi_2)B(\alpha) - K(\alpha) \]

(54)

Using equation (49) we can rewrite the payoff function in the following way

\[ U_2(D, \alpha) = (1 - t) \left\{ \frac{1}{2} (1 - \pi_2)\tilde{R}(\alpha) - \frac{(1 - s)(1 + r)}{2 - s} D \right\} + \frac{1 - s + t}{2 - s} - \frac{D - I + (1 - \pi_2)B(\alpha) - K(\alpha)}{2 - s} \]

(55)

The investor’s maximization problem is characterized by the following two first order conditions.

\[ \frac{dU}{d\alpha} = (1 - t) \left\{ \frac{1}{2} (1 - \pi_2) \frac{d\tilde{R}}{d\alpha} - \frac{1 - s}{2 - s} D \frac{dr}{d\tilde{R}} + (1 - \pi_2)B' - K' \right\} = 0 \]

(57)

\[ \frac{dU}{dD} = -(1 - t) \left\{ \frac{1 - s}{2 - s} (1 + r) + \frac{dr}{dD} \right\} + \frac{1 - s + t}{2 - s} = 0 \]

(58)

Using (51), we can rewrite the first order condition for the optimal \( D \) in the following way

\[ \frac{dU}{dD} = -(1 - \pi)(1 - t) \frac{1 - s}{2 - s} \left[ 1 + r + \frac{r(1 + r)}{(1 - r)D} \right] + \frac{1 - s + t}{2 - s} = 0 \]

(59)

Rearranging yields

\[ \frac{1 - s + t}{2 - s} = (1 - t) \frac{1 - s}{2 - s} \frac{1 + r}{(1 - r)} \]

(60)

We can use this condition to solve for \( r_2 \) and \( 1 + r_2 \):

\[ r_2 = \frac{(2 - s)t}{2(1 - s) + st} \quad 1 + r_2 = \frac{2(1 - s + t)}{2(1 - s) + st} \]

(61)

Inserting this in (49), we can solve for

\[ D_2 = (1 - \pi_2) \frac{\tilde{R}}{2} \frac{2(1 - s) + st}{(1 - s + t)^2} = D_1 \]

(62)

Using the solutions for \( r_2 \) and \( D_2 \), we can finally determine the investor’s payoff

\[ U_2 = (1 - \pi_2) \left\{ \frac{\tilde{R}}{2} \frac{1 - s + st}{1 - s + t} + B(\alpha) \right\} - K(\alpha) - I \]

(63)

Q.E.D.
Proof of Result 4

In Result 3 we have seen that the optimal debt levels and the investor’s payoff are the same in both cases, expropriation and creeping expropriation. We now determine the comparative statics with respect to the local taxation rate $\pi_i$, with $i = \{1, 2\}$.

To see how the optimal debt level reacts to changes in $\pi_1$, consider the optimal debt level as characterized in (43) and (62).

$$\frac{dD^*_i}{d\pi_i} = -\frac{1}{2} \bar{R}t \left( \frac{2(1-s)+st}{(1-s+t)^2} \right) < 0$$ (64)

To determine how the ownership ratio $\alpha$ reacts to changes in $\pi_i$, we use the first order condition of (44) or (63) that implicitly defines $\alpha^*$

$$(1-\pi_i) \left[ \left( \frac{1-s+st}{1-s+t} \right) \frac{1}{2} \frac{d\bar{R}}{d\alpha} + B' \right] - K' = 0$$ (65)

From this we find, using the implicit function theorem

$$\frac{d\alpha^*}{d\pi_i} = -\frac{-\left( \frac{1-s+st}{1-s+t} \right) \frac{1}{2} \frac{d\bar{R}}{d\alpha} - B'}{(1-\pi_i) \left( \frac{1-s+st}{1-s+t} \right) \frac{d^2\bar{R}}{d\alpha^2} + (1-\pi_i) \frac{B''}{\bar{R}} - K''} < 0$$ (66)

where the negative sign of the nominator is due to the fact that the first order condition (65) needs to be satisfied.

Q.E.D.

Proof of Result 5

Recall that in case of confiscatory taxation the interest rate is implicitly determined by the same break even condition for the bank as in the base line model:

$$\int_{(1+r)D}^{\bar{R}} (1+r)D \frac{1}{\bar{R}} dR + \int_0^{(1+r)D} sR \frac{1}{\bar{R}} dR = D$$ (67)

This implies the same implicit function and hence the same conditions for the interest rate as above.

$$2r\bar{R} - (2-s)(1+r)^2D = 0$$ (68)

$$\frac{dr}{dD} = \frac{r(1+r)}{(1-r)D}$$ (69)
and
\[
\frac{dr}{dR} = \frac{-r(1+r)}{(1-r)R}
\]  
(70)

The investor maximizes
\[
U_3(D, \alpha) = \int_{(1+r)D}^{R(\alpha)} (1-t-\pi_3)[R - (1+r)D] \frac{1}{R(\alpha)} dR + D - I + B(\alpha) - K(\alpha)  
\]  
(71)

Using equation (68) we can rewrite the payoff function in the following way
\[
U_3(D, \alpha) = (1-t-\pi_3) \left[ \frac{1}{2} \bar{R}^2(\alpha) - \frac{(1-s)(1+r)}{2-s} D \right] + \frac{1-s+t+\pi_3}{2-s} D - I + B(\alpha) - K(\alpha).  
\]  
(72)

The investor’s maximization problem is characterized by the following two first order conditions.
\[
\frac{dU_3}{d\alpha} = (1-t-\pi_3) \left[ \frac{1}{2} \frac{d\bar{R}}{d\alpha} - \frac{1-s}{2-s} D \frac{dr}{d\alpha} \right] + B' - K' = 0  
\]  
(73)
\[
\frac{dU_3}{dD} = -(1-t-\pi_3) \frac{1-s}{2-s} \left[ (1+r) + \frac{dr}{dD} \right] + \frac{1-s+t+\pi_3}{2-s} = 0  
\]  
(74)

Using (69), we can rewrite the first order condition for the optimal \(D\) in the following way
\[
\frac{dU_3}{dD} = -(1-t-\pi_3) \frac{1-s}{2-s} \left[ 1 + r + \frac{r(1+r)}{(1-r)D} \right] + \frac{1-s+t+\pi_3}{2-s} = 0  
\]  
(75)

Rearranging yields
\[
\frac{1-s+t+\pi_3}{2-s} = (1-t-\pi_3) \frac{1-s}{2-s} \frac{1+r}{1-r}  
\]  
(76)

We can use this condition to solve for \(r_3\) and \(1+r_3\):
\[
r_3 = \frac{(2-s)(t+\pi_3)}{2(1-s) + s(t+\pi_3)}  
\]
\[
1 + r_3 = \frac{2(1-s+t+\pi_3)}{2(1-s) + s(t+\pi_3)}  
\]  
(77)

Inserting this in (68), we can solve for
\[
D_3^* = \frac{\bar{R}}{2(1+\pi_3)} \frac{2(1-s) + s(t+\pi_3)}{(1-s+t+\pi_3)^2}  
\]  
(78)
Using the solutions for $r_3$ and $D_3$, we can finally determine the investor’s payoff

$$U_3 = \frac{\bar{R}}{2} \frac{1 - s + s(t + \pi_3)}{1 - s + t + \pi_3} - I + (1 - \pi_3)B(\alpha) - K(\alpha)$$

(79)

Q.E.D.

Proof of Result 6

To see how the optimal debt level reacts to changes in $\pi_3$, consider the optimal debt level as characterized by (78):

$$\frac{dD_3^*}{d\pi_3} = \frac{\bar{R}}{2} \frac{2(1 - s) + s(t + \pi_3)}{(1 - s + t + \pi_3)^2} + (t + \pi_3) \frac{(1 - s + t + \pi_3)^2 s - 2(1 - s) + s(t + \pi_3)(1 - s + t + \pi_3)}{(1 - s + t + \pi_3)^4}
\frac{\bar{R}(1 - s)^2}{(1 - s + t + \pi_3)^3} > 0$$

To determine how the ownership ratio $\alpha$ reacts to changes in $t$, consider the first order condition of (79) that implicitly defines $\alpha^*_3$:

$$\left[1 - s + s(t + \pi_3)\right] \frac{1}{2} \frac{d\bar{R}}{d\alpha} + (1 - \pi_3)B' - K' = 0$$

(81)

From this we find, using the implicit function theorem,

$$\frac{d\alpha^*_3}{d\pi_3} = -\frac{\left[1 - s + s(t + \pi_3)\right] \frac{1}{2} \frac{d^2\bar{R}}{d\alpha^2} + (1 - \pi_3)B'' - K''}{\left[1 - s + s(t + \pi_3)\right] \frac{1}{2} \frac{d\bar{R}}{d\alpha} + (1 - \pi_3)B' - K'}$$

(82)

Note that the sign of $\frac{d\alpha^*_3}{d\pi_3}$ depends on the sign of $\frac{d\bar{R}}{d\alpha}$. Using equation (81), we find that $R' > 0$ if $B' < K'$ in the relevant parameter range, and hence $\frac{d\alpha^*_3}{d\pi_3} < 0$ if $B' < K'$.

Q.E.D.

Proof of Result 7

Consider

$$\frac{d\alpha^*_1}{d\pi_1} = \frac{(1 - s + st) \frac{1}{2} \frac{d\bar{R}}{d\alpha} + B'}{(1 - \pi_1) \left[1 - s + st\right] \frac{1}{2} \frac{d^2\bar{R}}{d\alpha^2} + B''} - K'' < 0$$

(83)

and

$$\frac{d\alpha^*_3}{d\pi_3} = \frac{\left[1 - s + s(t + \pi_3)\right] \frac{1}{2} \frac{d\bar{R}}{d\alpha} + B'' - K''}{\left[1 - s + s(t + \pi_3)\right] \frac{1}{2} \frac{d^2\bar{R}}{d\alpha^2} + B'' - K''} < 0 \text{ if } \bar{R} > 0$$

(84)
To see that $\frac{\partial \alpha_{1/2}}{\partial \pi_{1/2}} < \frac{\partial \alpha_{3}}{\partial \pi_{3}}$, it is sufficient to show that nominator of $\frac{|\partial \alpha_{1/2}|}{|\partial \pi_{1/2}|}$ is larger than the nominator of $\frac{|\partial \alpha_{3}|}{|\partial \pi_{3}|}$ and the denominator is smaller than the respective denominator. Simple rearranging of the respective equations prove that this is indeed the case. Q.E.D.